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# On a generalized nonlinear functional equation

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ABSTRACT

The paper deals with the functional equation

$$f(x) = F(f(u(x)), f(v(x)))$$

under some special assumptions concerning the given functions $u$, $v$ and $F$. Our main result extends some results in the literature.



## 1. Introduction and preliminaries

The aim of this note is to study the solutions of the functional equation

$$f(x) = F(f(u(x)), f(v(x))) \tag{1.1}$$

with real $x$ and continuous functions $u$ and $v$, which is a generalization of the functional equation

$$f(x) = \frac{1}{2}\left(f\left(\frac{x}{2}\right) + f\left(\frac{x+1}{2}\right)\right) \tag{1.2}$$

[1–3] with the well-known solution

$$f(x) = \cot(\pi x). \tag{1.3}$$

Concerning further linear equations (1.1) with constant coefficients, cf. paper [1] by Baron and Jarczyk as well as the references therein. Some related results can be found in survey [4].

In the proof of the first theorem we develop a method, which is called in book [5, p. 129] by Aigner and Ziegler *coup de grâce*, or Herglotz trick [5, pp.127,128] in connection with Eq. (1.2).

In what follows $I$ denotes a real compact interval, and we use the decomposition $I = I_1 \cup I_2$ into nondegenerated subintervals, where the intersection $I_1 \cap I_2$ can be also an interval (not a point only).

Moreover, we need the following simple auxiliary result. We define $\mathcal{F}_n$ as the set of all $2^n$ compositions of $u$ and $v$, which are self-maps of the same set, i.e.

$$\mathcal{F}_1 = \{u, v\},$$
$$\mathcal{F}_2 = \{u \circ u, u \circ v, v \circ u, v \circ v\},$$
$$\mathcal{F}_3 = \{u \circ u \circ u, u \circ u \circ v, u \circ v \circ u, u \circ v \circ v, v \circ u \circ u, v \circ u \circ v, v \circ v \circ u, v \circ v \circ v, \}$$

etc.

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