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Banach operator pairs and common fixed points in hyperconvex metric spaces

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ABSTRACT

The purpose of this paper is to establish DeMarr's well-known theorem for an arbitrary family of symmetric Banach operator pairs in hyperconvex metric spaces without the compactness assumption. We also give necessary and sufficient criteria for the existence of a common fixed point of a semigroup of isometric mappings. As an application, several results on the invariant best approximation are proved.

1. Introduction

The celebrated result of the existence of a common fixed point for a nonexpansive commutative family was first established by DeMarr [1] under the assumption that C is a compact convex subset of a normed space X. In 1965, Browder [2] obtained the corresponding result under the assumption that C is a bounded, closed and convex subset of a uniformly convex Banach space X. In 1992, Khamsi et al. [3] established the above mentioned results for a finite as well as an arbitrary commutative family of maps in hyperconvex metric spaces. Recently, Espinola and Hussain [4] proved DeMarr's theorem in uniformly convex metric spaces of type (T). More recently, Chen and Li [5] have introduced the concept of symmetric Banach operator pairs and extended DeMarr's result to the family of these operators.

In this paper we establish DeMarr's result for an arbitrary family of symmetric Banach operator pairs in hyperconvex metric spaces without the compactness assumption. We also give necessary and sufficient criteria for a common fixed point of a semigroup of isometric mappings. As an application, several results on the invariant best approximation are proved.

The notion of hyperconvexity is due to Aronszajn and Panitchpakdi [6] who proved that a hyperconvex space is a nonexpansive absolute retract, i.e., it is a nonexpansive retract of any metric space in which it is isometrically embedded. The interest in these spaces goes back to the results of Sine [7] and Soardi [8] who proved independently that the fixed point property for nonexpansive mappings holds in bounded hyperconvex spaces. Since then many interesting results have appeared in the setting of hyperconvex spaces (see [9–11]).

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