Nonlinear Analysis 74 (2011) 3440–3444

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

A unified approach to the asymptotic almost-equivalence of evolution systems without Lipschitz conditions

Felipe Álvarez [a], Juan Peypouquet [b,*]

[a] CMM (CNRS UMI 2807), Departamento de Ingeniería Matemática, Universidad de Chile, Av. Blanco Encalada 2120, Santiago, Chile

[b] Departamento de Matemática, Universidad Técnica Federico Santa María, Av. España 1680, Valparaíso, Chile

ARTICLE INFO

Article history:
Received 17 January 2011
Accepted 24 February 2011
Accepting Editor: S. Ahmad

MSC:
primary 34E10
secondary 34G25
37L05
47H20
47J35

Keywords:
Evolution systems
Asymptotic behavior
Almost-orbits
Asymptotic almost-equivalence

ABSTRACT

We study the asymptotic behavior of almost-orbits of evolution systems in Banach spaces without any continuity assumptions on either the space or the time dependence. We establish, in a unified framework, standard convergence, ergodic convergence and almost-convergence of almost-orbits for both the weak and the strong topologies on the basis of the analogue behavior of orbits.

1. Introduction and preliminaries

Let C be a nonempty Borel subset of a Banach space $(X, \|\cdot\|)$. An *evolution system* on C is a two-parameter family $U = \{U(t,s) \mid t \geq s \geq 0\}$ of possibly nonlinear maps from C into itself satisfying:

(i) $\forall t \geq 0, \forall x \in C, U(t,t)x = x$.
(ii) $\forall t \geq s \geq r \geq 0, \forall x \in C, U(t,s)U(s,r)x = U(t,r)x$.

The evolution system U is *Lipschitz* if there exists a constant $L > 0$ such that $\|U(t,s)x - U(t,s)y\| \leq L\|x - y\|$ for all $t \geq s \geq 0, x, y \in C$. An operator semigroup $T = \{T(t)\}$ defines an *autonomous* evolution system via $U(t,s) = T(t-s)$.

An *orbit* of U is a function $u : [0,\infty) \to C$ such that

$$\forall t \geq 0, \forall h \geq 0, \quad u(t+h) = U(t+h,t)u(t).$$

More generally, a function $u \in L^\infty_{\mathrm{loc}}(0,\infty; C)$ is an *almost-orbit* of U if

$$\lim_{t\to\infty} \sup_{h\geq 0} \|u(t+h) - U(t+h,t)u(t)\| = 0. \tag{1}$$

* Corresponding author.
E-mail addresses: falvarez@dim.uchile.cl (F. Álvarez), juan.peypouquet@usm.cl, jpeypou@gmail.com (J. Peypouquet).
URLs: http://www.dim.uchile.cl/~falvarez/ (F. Álvarez), http://docencia.mat.utfsm.cl/~jpeypou/ (J. Peypouquet).

0362-546X/$ – see front matter
doi:10.1016/j.na.2011.02.030