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Complex extreme points and complex convexity in Orlicz–Bochner function spaces[☆]

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ABSTRACT

It is well known that extreme points which are connected with strict convexity of the whole spaces, are the most basic and important geometric points in geometric theory of Banach spaces. In this paper, criteria for complex extreme points, complex strict convexity and complex uniform convexity in Orlicz–Bochner function spaces are given.

1. Introduction

Let $(X, \|\cdot\|_X)$ be a Banach space over the complex field C, let i be the complex number satisfying $\mathrm{i}^2 = -1$, and let $B(X)$ and $S(X)$ be the closed unit ball and the unit sphere of X, respectively. In what follows, N and R denote the set of natural numbers and the set of real numbers, respectively.

In the early 1980s, many papers in the area of the geometry of Banach spaces were devoted to the complex geometry of complex Banach spaces. It is well known that the complex geometric properties of complex Banach spaces have applications in various branches, among others in Harmonic Analysis, Operator Theory, Banach Algebras, C^*-Algebras, Differential Equation, Quantum Mechanics and Hydrodynamics Theory. It is also known that extreme points which are connected with strict convexity of the whole spaces, are the most basic and important geometric points in geometric theory of Banach spaces.

Before starting with our results, we need to recall some notions.

A point $x \in S(X)$ is said to be a complex extreme point of $B(X)$ if for every non-zero $y \in X$ there holds $\max_{|\lambda|=1} \|x+\lambda y\|_X > 1$. A complex Banach space X is said to be complex strictly convex if every element of $S(X)$ is a complex extreme point of $B(X)$. The set of all complex extreme points of the unit ball $B(X)$ will be denoted by $C - \mathrm{Ext}\, B(X)$. A complex Banach space X is said to be complex uniformly convex if for any $\varepsilon > 0$, there exists $\delta > 0$ such that $\sup_{|\lambda|\le 1} \|x + \lambda y\|_X \ge 1 + \delta$ whenever $x \in S(X)$ and $\|y\| \ge \varepsilon$ (see [1,2]).

A map $\Phi : R \to [0, \infty]$ is said to be an Orlicz function if Φ is even, convex, vanishing at zero and continuous on the whole of R^+. Let $p(u)$ be the right-hand side derivative of $\Phi(u)$. For every Orlicz function Φ, we define its complementary

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