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Global existence for nonlocal MEMS

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ABSTRACT

We prove the existence of solutions for a nonlocal equation arising from the mathematical modeling of MicroElectroMechanicalSystems (MEMS). The existence result, obtained within a suitable Implicit Function Theorem framework, is established under rather general boundary conditions and for bounded domains whose diameter is fairly small.

1. Introduction

Recently, a lot of attention has been devoted to the study of mathematical models which describe, with different levels of accuracy, the so-called electrostatic actuation in *MicroElectroMechanicalSystems (MEMS)*; see [1,2] and the references therein. Motivated by the mathematical interest which continues a field of research, initiated by Mignot–Puel in [3], on nonlinear problems involving nonlinearities which develop singularities; see also [4] for applications to conformal geometry, here we consider a wide class of nonlocal problems, namely the following

$$\begin{cases} \alpha\Delta^2 u = \left(\beta\displaystyle\int_\Omega |\nabla u|^2\,\mathrm{d}x + \gamma\right)\Delta u + \dfrac{\lambda f(x)}{(1-u)^\sigma\left(1+\chi\int_\Omega \frac{\mathrm{d}x}{(1-u)^\sigma}\right)}, & x \in \Omega \\ u = \Delta u - d u_\nu = 0, \quad x \in \partial\Omega,\ d \geq 0 \\ 0 < u < 1, \quad x \in \Omega \end{cases} \tag{1}$$

which for $\sigma \geq 2$ is a natural extension of the nonlocal MEMS problem which has been proposed in [5,1] in the case of a Coulomb potential, namely $\sigma = 2$. Here $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $u : \Omega \longrightarrow \mathbb{R}$ is the unknown profile of the deflecting MEMS plate, $f : \Omega \longrightarrow \mathbb{R}^+$ is a bounded function which carries dielectric properties of the material, $\lambda \geq 0$ is the drop voltage between the ground plate and the deflecting plate and for positive parameters $\alpha, \beta, \gamma, \chi$ which

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