Nonlinear Analysis 74 (2011) 2462–2467

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Existence and uniqueness of a solution for a two dimensional nonlinear inverse diffusion problem

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ARTICLE INFO

Article history:
Received 22 February 2010
Accepted 4 December 2010

MSC:
35R30
35K57

Keywords:
Inverse problem
Nonlinear diffusion problem
Square porous medium

ABSTRACT

The problem of identifying the coefficient in a square porous medium is considered. It is shown that under certain conditions of data f, g, and for a properly specified class $\mathcal{A}$ of admissible coefficients, there exists at least one $a \in \mathcal{A}$ such that (a, u) is a solution of the corresponding inverse problem.

1. Introduction

For $Q_T = \Omega \times (0, T)$, a domain in $\mathbb{R}^3$, where $\Omega = (0, 1) \times (0, 1)$ is the open unit square in $\mathbb{R}^2$, and $T > 0$, we consider the following nonlinear diffusion problem

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[a(u)\frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[a(u)\frac{\partial u}{\partial y}\right] \quad \text{in } Q_T, \tag{1.1a}$$

$$u(x, y, 0) = 0, \quad (x, y) \in \overline{\Omega} \tag{1.1b}$$

$$-a(u(0, y, t))\frac{\partial u}{\partial t}(0, y, t) = g(y, t), \quad y \in [0, 1],\ t \in [0, T], \tag{1.1c}$$

$$\frac{\partial u}{\partial x}(1, y, t) = 0, \quad y \in [0, 1],\ t \in [0, T], \tag{1.1d}$$

$$\frac{\partial u}{\partial y}(x, 0, t) = 0, \quad x \in [0, 1],\ t \in [0, T], \tag{1.1e}$$

$$\frac{\partial u}{\partial y}(x, 1, t) = 0, \quad x \in [0, 1],\ t \in [0, T], \tag{1.1f}$$

$$u(0, y, t) = f(y, t), \quad y \in [0, 1],\ t \in [0, T], \tag{1.1g}$$

where $g(y, t)$ and $f(y, t)$ are known functions and $a(u)$ and $u(x, y, t)$ are unknown functions.

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0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.12.001