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Best and coupled best approximation theorems in abstract convex metric spaces

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ABSTRACT

In this paper, we first give a best approximation theorem in abstract convex metric spaces. As applications, we then derive some best and coupled best approximations and coupled coincidence point results in normed spaces and hyperconvex metric spaces.

1. Introduction and preliminaries

The well known best approximation theorem of Fan [1] has been of great importance in nonlinear analysis, approximation theory, minimax theory, game theory, fixed point theory, and variational inequalities. Interesting extensions have been given by several authors and a variety of applications, mostly in fixed point theory and approximation theory, have also given by many, see [2–11] and references therein. Recently, Mitrović [11], proved a coupled best approximation theorem in normed spaces and derived some coupled coincidence and coupled fixed point results. In this paper, we first introduce a coupled coincidence point for set-valued maps and give a best approximation theorem in abstract convex metric spaces. As applications, we then derive some best and coupled best approximations and coincidence point results in normed spaces and hyperconvex metric spaces.

In the rest of this section we recall some definitions and theorems which are used in the next section. Let X and Y be topological spaces with $A \subseteq X$ and $B \subseteq Y$. Let $F : X \multimap Y$ be a set-valued map with nonempty values. The image of A under F is the set $F(A) = \bigcup_{x \in A} F(x)$ and the inverse image of B under F is

$$F^{-}(B) = \{x \in X : F(x) \cap B \neq \emptyset\}.$$

Now F is said to be:

(i) upper semicontinuous (usc, for short), if for each closed set $B \subseteq Y$, $F^{-}(B)$ is closed in X;
(ii) lower semicontinuous (lsc, for short), if for each open set $B \subseteq Y$, the set $F^{-}(B)$ is open;
(iii) continuous if, F is both lsc and usc;
(iv) closed, if its graph $\mathrm{Gr}(F) = \{(x, y) \in X \times Y : y \in F(x)\}$ is a closed set in the product space $X \times Y$.

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