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Note on "Coupled fixed point theorems for contractions in fuzzy metric spaces"

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ABSTRACT

In the paper "Coupled fixed point theorems for contractions in fuzzy metric spaces" by Sedghi et al. [S. Sedghi, I. Altun, N. Shobec, Coupled fixed point theorems for contractions in fuzzy metric spaces, Nonlinear Analysis 72 (2010) 1298–1304], a coupled common fixed point result was presented. However, our purpose is to show that this result and its proof are false. We give a counterexample and also explain how to correct this result. As a modification, we state and prove a coupled fixed point theorem under some hypotheses of fuzzy metric and t-norm.

1. Introduction and preliminaries

In [1], the authors proved a coupled common fixed point theorem for contractive mappings in complete fuzzy metric spaces defined by George and Veeramani. The main result in [1] is Theorem 2.5. Unfortunately, it is false. In Section 2, we give a counterexample. In Section 3, we show that the process of the proof is false and also explain how to correct this result. In Section 4, as a modification, we state and prove a coupled fixed point theorem under some hypotheses concerning fuzzy metric and t-norm.

For the reader's convenience, we restate some definitions and the main result in [1] as follows.

Definition 1.1 (*[1]*)**.** A binary operation $* : [0, 1]^2 \to [0, 1]$ is called a continuous t-norm if the following conditions are satisfied: $*$ is associative and commutative; $*$ is continuous; $a * 1 = a$ for all $a \in [0, 1]$; $a * b \le c * d$, whenever $a \le c$ and $b \le d$, for each $a, b, c, d \in [0, 1]$.

Definition 1.2 (*[1]*)**.** The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary non-empty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions, for each $x, y \in X$ and $t, s > 0$,

(FM-1) $M(x, y, t) > 0$,
(FM-2) $M(x, y, t) = 1$ if and only if $x = y$,
(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(FM-5) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous.

Definition 1.5 (*[1]*)**.** Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$. A sequence $\{x_n\}$ in X is called the Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$,

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