Nonlinear Analysis 74 (2011) 6116–6120

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

An implicit iteration process for nonexpansive semigroups

Duong Viet Thong

Faculty of Economics Mathematics, National Economics University, 207 Giai Phong St., Hai Ba Trung District, Hanoi City, Viet Nam

ARTICLE INFO

Article history:
Received 17 September 2010
Accepted 31 May 2011
Communicated by Ravi Agarwal

Keywords:
Nonexpansive semigroup
Common fixed point
Opial's condition
Implicit iteration process

ABSTRACT

Let C be a closed convex subset of a Banach space E. Let $\{T(t) : t \geqslant 0\}$ be a strongly continuous semigroup of nonexpansive mappings on C such that $\bigcap_{t\geqslant 0} F(T(t)) \neq \emptyset$. Let $\{\alpha_n\}$ and $\{t_n\}$ be sequences of real numbers satisfying appropriate conditions, then for arbitrary $x_0 \in C$, the Mann type implicit iteration process $\{x_n\}$ given by $x_n = \alpha_n x_{n-1} + (1 - \alpha_n)T(t_n)x_n$, $n \geqslant 0$, weakly (strongly) converges to an element of $\bigcap_{t\geqslant 0} F(T(t))$.

1. Introduction

Let E be a real Banach, E^* is the dual space of E, C is a nonempty closed convex subset of E. $J : E \to 2^{E^*}$ is the normalized duality mapping defined by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|.\|f\|, \|x\| = \|f\|\}, \quad x \in E.$$

Let $T : C \to C$ be a nonexpansive mapping (i.e., $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$). We use $F(T)$ to denote the set of fixed points of T, i.e., $F(T) := \{x \in C : x = Tx\}$. We know that $F(T)$ is nonempty if E is a reflexive Banach space with the Opial condition and C is a nonempty closed convex and bounded subset of E. Fix $u \in C$. Then for each $\alpha \in (0, 1)$, there exists a unique point $x_\alpha \in C$ satisfying $x_\alpha = \alpha u + (1 - \alpha)Tx_\alpha$ because the mapping $x \mapsto \alpha u + (1 - \alpha)Tx$ is contractive. In 1967, Browder [1] considered an implicit iteration for approximating fixed points of a nonexpansive mapping in a Hilbert space.

Theorem 1.1 (*Browder [1]*)**.** *Let C be a closed convex subset of a Hilbert space H and let T be a nonexpansive mapping on C with a fixed point. Let α_n be a sequence of $(0, 1)$ converging to 0. Fix $u \in C$ and define a sequence x_n by*

$$x_n = \alpha_n u + (1 - \alpha_n)Tx_n, \quad n \in \mathbb{N}. \tag{1.1}$$

Then $\{x_n\}$ converges strongly to the element of $F(T)$ nearest to u.

Let $\{T(t) : t \geqslant 0\}$ be a strongly continuous semigroup of nonexpansive mapping on a closed convex subset C of a Banach space E, i.e.,

(1) for each $t \geqslant 0$, $T(t)$ is a nonexpansive mapping on C;
(2) $T(0)x = x$ for all $x \in C$;
(3) $T(s + t) = T(s) \circ T(t)$ for all $s, t \geqslant 0$;
(4) for each $x \in E$, the mapping $T(.)x$ from $\mathbb{R}_+$ into C is continuous.

We put $F(T) = \bigcap_{t\geqslant 0} F(T(t))$. We know that $F(T)$ is nonempty if C is a nonempty closed convex and bounded subset in a uniformly convex Banach space E [2]. In 1998, Shioji and Takahashi [3] proved the following result.

Theorem 1.2 (*Shioji and Takahashi [3]*)**.** *Let C be a closed convex subset of a Hilbert space H. Let $\{T(t) : t \geqslant 0\}$ be a strongly continuous semigroup of nonexpansive mappings on C such that $F(T) \neq \emptyset$. Let $\{\alpha_n\}$ and $\{t_n\}$ be sequences of real numbers*

E-mail address: thongduongviet@gmail.com.

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.05.090