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# Blowup for the Euler and Euler–Poisson equations with repulsive forces

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ABSTRACT

In this paper, we study the blowup of the $N$-dim Euler or Euler–Poisson equations with repulsive forces, in radial symmetry. We provide a novel integration method to show that the non-trivial classical solutions $(\rho, V)$, with compact support in $[0, R]$, where $R > 0$ is a positive constant and in the sense which $\rho(t, r) = 0$ and $V(t, r) = 0$ for $r \geq R$, under the initial condition

$$H_0 = \int_0^R rV_0 \mathrm{d}r > 0, \tag{1}$$

blow up on or before the finite time $T = R^3/H_0$ for pressureless fluids or $\gamma > 1$.

The main contribution of this article provides the blowup results of the Euler ($\delta = 0$) or Euler–Poisson ($\delta = 1$) equations with repulsive forces, and with pressure ($\gamma > 1$), as the previous blowup papers (Makino et al., 1987 [18], Makino and Perthame, 1990 [19], Perthame, 1990 [20] and Chae and Tadmor, 2008 [24]) cannot handle the systems with the pressure term, for $C^1$ solutions.



## 1. Introduction

The isentropic Euler ($\delta = 0$) or Euler–Poisson ($\delta = \pm 1$) equations can be written in the following form:

$$\begin{cases} \rho_t + \nabla \cdot (\rho u) = 0, \\ \rho[u_t + (u \cdot \nabla)u] + \nabla P = \rho \nabla \Phi, \\ \Delta \Phi(t, x) = \delta \alpha(N) \rho, \end{cases} \tag{2}$$

where $\alpha(N)$ is a constant related to the unit ball in $R^N$: $\alpha(1) = 1$, $\alpha(2) = 2\pi$ and $\alpha(3) = 4\pi$. And as usual, $\rho = \rho(t, x) \geq 0$ and $u = u(t, x) \in \mathbf{R}^N$ are the density and the velocity respectively. $P = P(\rho)$ is the pressure function. The $\gamma$-law can be applied on the pressure term $P(\rho)$, i.e.

$$P(\rho) = K\rho^\gamma, \tag{3}$$

which is a common hypothesis. If the parameter is set as $K > 0$, we call the system *with pressure*; if $K = 0$, we call it *pressureless*. The constant $\gamma = c_P/c_v \geq 1$, where $c_P$, $c_v$ are the specific heats per unit mass under constant pressure and constant volume respectively, is the ratio of the specific heats, that is, the adiabatic exponent in Eq. (3). In particular, the fluid is called isothermal if $\gamma = 1$. If $K > 0$, we call the system with pressure; if $K = 0$, we call it pressureless.

In the above systems, the self-gravitational potential field $\Phi = \Phi(t, x)$ is determined by the density $\rho$ itself, through the Poisson equation $(2)_3$.

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