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Note on a general Palais–Smale condition

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ABSTRACT

In this paper, we establish a qualitative deformation theorem for a locally Lipschitz function satisfying a general Palais–Smale type condition. Then we show that, with some assumptions, this compactness condition implies the coercivity of the function.

1. Introduction

It is known that the Palais–Smale condition provides the existence of a critical point for many variational problems (see [1–4]). Its local version, imposed by practical problems and introduced in [5], is successfully applied. A compactness condition of the Palais–Smale type, the so-called Cerami condition, introduced in [6,7], is weaker than the Palais–Smale condition, but the most important implications are retained (see [8]). Other generalizations appear in [9–11]. In order to extend the non-smooth critical point theory of [9,12], a more general compactness condition, which includes all the above conditions, is introduced in [13].

In this paper, we establish a qualitative deformation theorem (see [14–16] for the smooth case) for a locally Lipschitz function satisfying the general compactness condition of [13]. We also prove that, with some assumption, this compactness condition implies the coercivity of the function, generalizing some classical results of [9,12,14,17–22].

2. Preliminaries

Let X be a Banach space with the dual X^* and the duality pairing $\langle\cdot,\cdot\rangle$. Let $f : X \to \mathbf{R}$ be a locally Lipschitz function. By $f^\circ(x, h)$ we denote the generalized directional derivative at $x \in X$ in the direction $h \in X$.

Recall that the generalized gradient, in the sense of Clarke, is defined as follows:

$$\partial f(x) = \{x^* \in X^* | \langle x^*, h\rangle \leq f^\circ(x, h), \text{ for any } h \in X\}.$$

It is known that $\partial f(x)$ is a non-empty, convex and w^*-compact set; see [23]. Remark that $f^\circ(x, h) = \max\{\langle x^*, h\rangle | x^* \in \partial f(x)\}$, for all $h \in X$. Moreover, if f is on C^1-class on X, then $\partial f(x) = \{df(x)\}$. If $0 \in \partial f(x)$, then we say that $x \in X$ is a critical point

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