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The existence of a weighted mean for almost periodic functions

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ABSTRACT

In a recent paper by Liang et al. (2010) [1], the original question which is that of the existence of a weighted mean for almost periodic functions was raised. In particular, they showed through an example that there exist weights for which a weighted mean for almost periodic functions may or may not exist. In this note we give some sufficient conditions which do guarantee the existence of a weighted mean for almost periodic functions, which will then coincide with the classical Bohr mean. Moreover, we will show that under those conditions, the corresponding weighted Bohr transform exists.

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1. Introduction

Let $\mathbb{U}$ denote the collection of all functions (weights) $\mu : \mathbb{R} \mapsto (0, \infty)$, which are locally integrable over $\mathbb{R}$ such that $\mu > 0$ almost everywhere. Throughout the rest of this note, if $\mu \in \mathbb{U}$ and $r > 0$, we then suppose that $Q_r := [-r, r]$, $Q_r + a := [-r + a, r + a]$, and that

$$\mu(Q_r) := \int_{Q_r} \mu(t)\mathrm{d}t.$$

In this note, we are exclusively interested in the weights, μ, for which $\mu(Q_r) \to \infty$ as $r \to \infty$. Consequently, we define the set of weights $\mathbb{U}_\infty$ by

$$\mathbb{U}_\infty := \left\{\mu \in \mathbb{U} : \lim_{r\to\infty} \mu(Q_r) = \lim_{r\to\infty} \int_{Q_r} \mu(t)\mathrm{d}t = \infty\right\}.$$

Suppose that $\mu \in \mathbb{U}_\infty$ and let $\mathbb{X}$ be a Banach space. If $f : \mathbb{R} \mapsto \mathbb{X}$ is a bounded continuous function, we define its *weighted mean*, if the limit exists, by

$$\mathcal{M}(f, \mu) := \lim_{r\to\infty} \frac{1}{\mu(Q_r)} \int_{Q_r} f(t)\mu(t)\mathrm{d}t.$$

In a recent paper by Liang et al. [1], the original question which is that of the existence of a weighted mean for almost periodic functions was raised. In particular, Liang et al. showed through an example that there exist weights $\mu \in \mathbb{U}_\infty$

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