Nonlinear Analysis 74 (2011) 4387–4391

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Fixed point theorem for α-nonexpansive mappings in Banach spaces

Koji Aoyama [a], Fumiaki Kohsaka [b,*]

[a] Department of Economics, Chiba University, Yayoi-cho, Inage-ku, Chiba-shi, Chiba 263-8522, Japan

[b] Department of Computer Science and Intelligent Systems, Oita University, Dannoharu, Oita-shi, Oita 870-1192, Japan

ARTICLE INFO

Article history:
Received 24 August 2010
Accepted 29 March 2011
Communicated by Ravi Agarwal

MSC:
primary 47H06
secondary 47H09
47H10

Keywords:
Firmly nonexpansive mapping
Fixed point theorem
Nonexpansive mapping
Nonspreading mapping
Hybrid mapping

ABSTRACT

We introduce the class of α-nonexpansive mappings in Banach spaces. This class contains the class of nonexpansive mappings and is related to the class of firmly nonexpansive mappings in Banach spaces. In addition, we obtain a fixed point theorem for α-nonexpansive mappings in uniformly convex Banach spaces.

1. Introduction

Let E be a Banach space and let C be a nonempty subset of E. We denote the fixed point set of a mapping $T: C \to E$ by $F(T)$. Following Bruck [1], we say that a mapping $T: C \to E$ is firmly nonexpansive if

$$\|Tx - Ty\| \leq \|r(x - y) + (1 - r)(Tx - Ty)\|$$

for all $r > 0$ and $x, y \in C$; see also [2]. It is obvious that every firmly nonexpansive mapping is nonexpansive, i.e., $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. See, for example, [2–7] for more information on firmly nonexpansive mappings.

In 2008, Kohsaka and Takahashi [8] studied the existence and approximation of fixed points of mappings of firmly nonexpansive type in Banach spaces. This is another generalization of firmly nonexpansive mappings in Hilbert spaces; see also [8–11]. Kohsaka and Takahashi [10] also introduced the class of nonspreading mappings in Banach spaces, which is wider than the class of mappings of firmly nonexpansive type, and obtained some fixed point theorems for these mappings. Takahashi [12] introduced the class of hybrid mappings in Hilbert spaces, which contains the class of firmly nonexpansive mappings in Hilbert spaces.

Recently, Aoyama et al. [13] introduced the class of λ-hybrid mappings in the Hilbert space setting (see Definition 2.1) and obtained a fixed point theorem and an ergodic theorem for these mappings. This class contains the classes of nonexpansive mappings, nonspreading mappings, and hybrid mappings in Hilbert spaces.

In this paper, we introduce the class of α-nonexpansive mappings in Banach spaces (see Definition 2.2) and generalize some of the results obtained in [13] from Hilbert spaces to more general Banach spaces. It should be noted that every firmly

* Corresponding author.
E-mail addresses: aoyama@le.chiba-u.ac.jp (K. Aoyama), f-kohsaka@oita-u.ac.jp (F. Kohsaka).

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.03.057