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Fixed point theorems in Boolean vector spaces

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ABSTRACT

In this paper, we obtain some fixed and common fixed point theorems in finite dimensional normed Boolean vector spaces. Our results extend and unify some known results.

1. Introduction

Fixed points of Boolean functions have numerous applications in the theory of error-correcting codes, applications to switching theory and to the relationship between the consistency of a Boolean equation, cryptography, convergence of some recursive parallel array processes in Boolean arrays, memory-efficient solution techniques in computer science etc. Fixed point theory of Boolean functions is an active area of research (see for instance [1,2] and references therein).

In [3], Subrahmanyam introduced the notion of Boolean and normed Boolean vector spaces and proved that a Boolean vector space is, in general, irreducible to a module over a Boolean ring. Further, he studied the basis and convergence in a normed Boolean vector space over a σ-complete Boolean algebra (see also [4]). On the other hand, Proinov [5] obtained a fixed point theorem on a complete metric space in a very general setting. Its subsequent extensions and generalizations appeared in [6,7].

In the present paper, we utilize the concept of normed Boolean vector spaces of Subrahmanyam [3] and extend certain results of [5,6] to finite dimensional normed Boolean vector spaces. Our approach in this paper is entirely algebraic.

2. Preliminary

The following definitions are essentially due to Subrahmanyam [3].

Definition 2.1 (*[3]*)**.** Let $V = (V, +)$ be an additive (Abelian) group and $(B, +, ., ')$ be a Boolean algebra, whose elements are denoted, respectively, by x, y, z and a, b, c (with or without indices); the "zero" of V and also the "null element" of B are both denoted by "0", while the "universal element" ($=0'$) of B will be denoted by "1".

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