Uniformly continuous composition operators in the space of functions of φ-variation with weight in the sense of Riesz

W. Aziz [a], A. Azocar [b], J. Guerrero [c,*], N. Merentes [d]

[a] *Universidad de los Andes, Escuela de Matemáticas, Mérida, Venezuela*
[b] *Universidad Nacional Abierta, Area de Matemática, Caracas, Venezuela*
[c] *Universidad Nacional Experimental del Táchira, Departamento de Matemática y Física, San Cristóbal, Venezuela*
[d] *Universidad Central de Venezuela, Escuela de Matemáticas, Caracas, Venezuela*

ARTICLE INFO

ABSTRACT

In this paper we prove that if a uniformly continuous Nemytskii operator maps a space of bounded variation with weight functions in the sense of Riesz into another space of the same type, its generator function is an affine function.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

Let I be an interval of $\mathbb{R}$, $(X, |\cdot|)$ a real normed space, C a closed subset of X, $(Y, |\cdot|)$ a real Banach space and $h : I \times C \longrightarrow Y$ a given function. Denote by X^I the set of all functions $f : I \longrightarrow X$ and by $H : X^I \longrightarrow Y^I$ the (Nemytskii or superposition) composition operator generated by the function h defined by

$$H(f)(\cdot) = h(\cdot, f(\cdot)).$$

In this paper, we prove that if H maps the space $RV_{\varphi,\lambda}(I, C)$ of functions of bounded φ-variation with weight in the sense of Riesz into the space $RV_{\psi,\lambda}(I, Y)$, and is uniformly continuous, then h, the generator function of the operator H, is an affine function in the second variable.

This generalizes the results of Chistyakov [1], where it is assumed that H is Lipschizian. The uniformly continuous composition operators were first considered in [2] for the space of differentiable functions and absolutely continuous functions, later in [3] for the space of the Hölder function, and in [4] for the space of bounded variation functions. Moreover, this result extend the main result obtained by [5].