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Uniformly continuous composition operators in the space of functions of φ -variation with weight in the sense of Riesz

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ABSTRACT

In this paper we prove that if a uniformly continuous Nemytskii operator maps a space of bounded variation with weight functions in the sense of Riesz into another space of the same type, its generator function is an affine function.

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1. Introduction

Let I be an interval of \mathbb{R} , $(X, |\cdot|)$ a real normed space, C a closed subset of X , $(Y, |\cdot|)$ a real Banach space and $h : I \times C \rightarrow Y$ a given function. Denote by X^I the set of all functions $f : I \rightarrow X$ and by $H : X^I \rightarrow Y^I$ the (Nemytskii or superposition) composition operator generated by the function h defined by

$$H(f)(\cdot) = h(\cdot, f(\cdot)).$$

In this paper, we prove that if H maps the space $RV_{\varphi, \lambda}(I, C)$ of functions of bounded φ -variation with weight in the sense of Riesz into the space $RV_{\psi, \lambda}(I, Y)$, and is uniformly continuous, then h , the generator function of the operator H , is an affine function in the second variable.

This generalizes the results of Chistyakov [1], where it is assumed that H is Lipschitzian. The uniformly continuous composition operators were first considered in [2] for the space of differentiable functions and absolutely continuous functions, later in [3] for the space of the Hölder function, and in [4] for the space of bounded variation functions. Moreover, this result extend the main result obtained by [5].

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