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Endpoints of multi-valued generalized weak contraction mappings

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ABSTRACT

Let (X, d) be a complete metric space, and let $T : X \to P_{\mathrm{cl,bd}}(X)$ be a multi-valued generalized weak contraction mapping. Then T has a unique endpoint if and only if T has the approximate endpoint property. Our results extend previous results given by Ćirić (1971) [15], Nadler (1969) [11], Daffer and Kaneko (1995) [9] and Amini-Harandi (2010) [8].

1. Introduction and preliminaries

Let (X, d) be a metric space and $P(X)$ denote the class of all subsets of X. Define

$$P_f(X) = \{A \subseteq X : A \neq \emptyset \text{ has property } f\}.$$

Thus $P_{\mathrm{bd}}(X)$, $P_{\mathrm{cl}}(X)$, $P_{\mathrm{cp}}(X)$ and $P_{\mathrm{cl,bd}}(X)$ denote the classes of bounded, closed, compact, and closed bounded subsets of X, respectively. Also $T : X \to P_f(X)$ is called a multi-valued mapping on X. A point x is called a fixed point of T if $x \in Tx$. Define $\mathrm{Fix}(T) = \{x \in X : x \in Tx\}$. An element $x \in X$ is said to be an endpoint of a multi-valued mapping T, if $Tx = \{x\}$. We denote the set of all endpoints of T by $\mathrm{End}(T)$. The investigation of endpoints of multi-valued mappings has received great attention in recent years (see [1–8]). A mapping $T : X \to X$ is said to be a weak contraction if there exists $0 \leq \alpha < 1$ such that

$$d(Tx, Ty) \leq \alpha N(x, y), \tag{1.1}$$

for all $x, y \in X$, where

$$N(x, y) := \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\right\}. \tag{1.2}$$

A multi-valued mapping $T : X \to P_{\mathrm{cl,bd}}(X)$ is said to be a weak contraction if there exists $0 \leq \alpha < 1$ such that

$$H(Tx, Ty) \leq \alpha N(x, y), \tag{1.3}$$

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