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Best proximity point theorems generalizing the contraction principle

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ABSTRACT

The primary objective of this article is to elicit some interesting extensions of the simple but powerful Banach's contraction principle to the case of non-self-mappings. In fact, due to the fact that best proximity point theorems fit the bill to this end, the proposed extensions are presented as best proximity point theorems for non-self proximal contractions which are more general than the notion of self-contractions.

1. Introduction

The famous Banach's contraction principle asserts that every contraction self-mapping on a complete metric space has a unique fixed point. The literature abounds with numerous extensions and variants of the aforesaid result which has far-reaching applications. However, almost all such results dilate upon the existence of a fixed point for self-mappings. Anomalously, this article explores some interesting generalizations of the contraction principle to the case of non-self-mappings. In fact, given non-empty closed subsets A and B of a complete metric space, a contraction non-self-mapping $S : A \longrightarrow B$ is improbable to have a fixed point. Eventually, it is quite natural to seek an element x such that $d(x, Sx)$ is minimum, which connotes that x and Sx are in close proximity to each other. Indeed, in light of the fact $d(x, Sx)$ is at least $d(A, B)$, best proximity point theorems accentuate the preceding viewpoint further to guarantee the existence of an element x such that $d(x, Sx)$ assumes the least possible value $d(A, B)$, thereby accomplishing the highest possible closeness between x and Sx. Such an element x for which $d(x, Sx) = d(A, B)$ is designated as a best proximity point of the non-self-mapping S. In other words, in the case that S has no fixed point, a best proximity point serves as an optimal approximate solution to the equation $Sx = x$, for the error involved $d(x, Sx)$ attains the global minimal value $d(A, B)$ for any best proximity point x. It is remarked that a best proximity point reduces to a fixed point if the underlying mapping is assumed to be a self-mapping. The preceding deliberation intrigues one to establish some best proximity point theorems that are natural generalizations of the contraction principle to the case of non-self-mappings. Indeed, this article elicits some best proximity point theorems for proximal contractions which are more general than the notion of self-contractions. One can refer to [1–4] for best proximity point theorems for some other kinds of contractions. However, such results do not extend the contraction principle.

It should be noted that best approximation theorems also furnish an approximate solution to the equation $Sx = x$ in the event that S has no fixed point. Indeed, a well-known best approximation theorem, due to Fan [5], asserts that if K is a non-empty compact convex subset of a Hausdorff locally convex topological vector space E and $T : K \longrightarrow E$ is a continuous mapping, then there exists an element x satisfying the condition that $d(x, Tx) = d(Tx, K)$. Subsequently, several authors, including Prolla [6], Reich [7], Sehgal and Singh [8,9], have derived extensions of this interesting result in many directions.

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