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Elastic stresses for 90° elbows under in-plane bending

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ABSTRACT

Using finite element analysis, this paper extends elastic stress solutions for 90° pipe elbows under in-plane bending, given in Marie et al. (2007) [1], to cases of mean pipe radius-to-thickness ratio up to 50. It is found that for 90° elbows an in-plane bending moment produces not only an axial membrane stress component but also axial and hoop bending stress components. Furthermore, the magnitudes of these stress components depend strongly on the mean radius-to-thickness ratio, the circumferential location and the longitudinal location. Maximum stresses tend to occur in the centre of the elbow at or near the crown.

1. Introduction

Determination of elastic stresses for piping components is important in fatigue and fracture assessments. For straight pipes under internal pressure, axial tension or bending, for example, elastic stresses can be determined from elementary solid mechanics. However, an elbow is commonly used in industrial plants for layout requirement. Many studies have been reported on plastic limit analyses of elbows (see for instance Refs. [2–18]) but those for elastic stress analysis are more limited. Determination of elastic stresses for elbows is not straightforward. For internal pressure, an analytical elastic stress solution was originally derived by Goodall [4]. Marie et al. [1] listed elastic solutions for an elbow, which are essentially the same as Goodall's result. More recently the present authors [19] found that elastic solutions in the centre of an elbow could be represented by Goodall's solution, but they varied with the longitudinal position within the elbow. Accordingly, the authors proposed stress solutions incorporating longitudinal variations based on detailed finite element (FE) analysis. For an elbow under in-plane bending, some design codes, such as the ASME BPV code [20] and the French RSE-M and RCC-MR codes [1], also provide elastic stress solutions. However, the solutions in the ASME and French codes differ and thus the accuracy of these solutions needs to be checked. Another issue to be resolved is that these existing solutions are mainly valid when the mean pipe radius-to-thickness ratio is less than 20. Although such a range can cover many critical piping components, piping systems often have elbows with a mean radius-to-thickness ratio larger than 20.

In this paper, systematic elastic FE analysis is performed to derive elastic stress solutions for 90° elbows under in-plane bending. Based on the FE results, the accuracy of existing solutions is checked and a new solution is presented to cover the case of mean radius-to-thickness ratios ranging from 20 to 50. Section 2 summarises existing elastic stress solutions for 90° elbows under in-plane bending. The new solution covering a wider range of the mean radius-to-thickness ratio is given in Section 3. The present work is discussed in Section 4 and concluded in Section 5.

2. Summary of existing elastic stress solutions for 90° elbows under in-plane bending

Fig. 1 depicts a 90° elbow subject to in-plane bending moment M_b, having outer pipe radius, r_o, inner pipe radius, r_i, mean pipe radius r_m, thickness, t, and bend radius, R. The circumferential position in the cross-section of the elbow is characterised by θ, where the values $\theta = -\pi/2$, 0 and $\pi/2$ correspond to the intrados, crown and extrados, respectively. The longitudinal position within the elbow is characterised by the angular variable ϕ with values $\phi = 0$ and $\pi/4$ corresponding to the junction and the elbow centre, respectively.

2.1. ASME BPV Sec III [20]

In Section III of the American Society of Mechanical Engineers (ASME) Boiler and Pressure Vessel (BPV) Code [20], elastic

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