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Nonlinear study of large deflection of simply supported piezoelectric layered-plate under initial tension

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ABSTRACT

The nonlinear problem of large deflection of a simply supported piezoelectric layered plate under initial tension is studied. The approach follows von Karman's plate theory for large deflection for a symmetrically layered isotropic case including a piezoelectric layer. The nonlinear governing equations are solved using a finite difference method, by taking the associated linear analytical solution as an initial guess in the numerical iteration procedure. The results for a nearly monolithic plate under a very low applied voltage are found to correlate well with available solutions for a single-layered case under pure mechanical loading and thus the present approach is validated. For three-layered plates made of typical silicon based materials, various initial tension and lateral pressure are considered, and different applied voltages up to a moderate magnitude are implemented. No edge effect was observed, in contrast to the cases of clamped plates in literature. In additions, varying the layer moduli seems to have an insignificant effect upon the structural responses of the layered plate. On the other hand, the piezoelectric effect tends to be apparent only in a low pretension condition. For a relatively large pretension, the effect of initial tension becomes dominant, yielding nearly unique solutions for the structural responses, regardless of the magnitudes of the applied voltage and the lateral pressure.

1. Introduction

A pressure-sensing device may often be operated under the condition of large deflection. Classical plate or lamination theory following the Kirchhoff assumption is no longer applicable in this case and hence a more advanced theoretical approach must be considered to evaluate their structural behaviors. In literature, there have been quite a few studies available that dealt with this type of problem. The earliest work may trace back to the approach discussed in the well-known textbook of Timoshenko and Woinowsky-Krieger [1]. Yet, it appeared to be due to Voorthuyzen and Bergveld [2], first, to solve for the problem of large deflection of a uniformly loaded circular diaphragm simulating a micro-sensing device under initial in-plane loading by the numerical method of finite difference. They included both bending and tensile stresses in formulating the problem of large deflection but only the results of deflection at the center of the plate were presented, including the study of effect of tensile force. A rigorous study of nearly the same problem was conducted by Sheplak and Dugundji [3] that presented both linear analytical solution and nonlinear numerical results. The influences of initial tension and lateral load upon the geometrical responses were illustrated and the behavior of a plate transforming from a plate mode to a membrane was thoroughly investigated. Based on a similar approach, the problem of large deflection of an annular plate with a rigid boss was later considered by Su et al. [4].

Following the previous works, the merit of a linear study was clearly illustrated that it may provide not only the primary insight for the problem but also a valuable initial prediction for solving the original nonlinear problem in a numerical procedure. Nevertheless, available studies thus far seem to have discussed mainly clamped-ended conditions and the other supporting cases i. e., the simply supported problems appear to have received relatively less attention in literature. One problem of simple support was examined by Lin et al. [5] and a simulation program was developed for investigating the sensitivity and behavior of linearity of a piezoresistive sensor, but only classical plate theory was utilized. A recent work considering geometrically nonlinear response of a circular plate under piezoelectric actuation was presented by Kapuria and Dumir [6], but only moderate large deflection was taken into account, neglecting the direct piezoelectric effect in problem formulation. In fact, although it may be close to a completely clamped situation, the edge around a typical micro-machined flat element is not completely clamped in reality. Solutions for simply supported problems of this type are thus desirable as they may provide an opposite bound for the structural responses.

In addition to large deflection, a micro-sensing device is very often subjected to initial tension. The cause may be due to the

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