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Determination of NSIFs and coefficients of higher order terms for sharp notches using finite element method

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A B S T R A C T

An overdeterministic method was used for calculating the Notch Stress Intensity Factors (NSIFs) as well as the coefficients of the higher order terms for structures containing sharp notches. The series solution of displacement fields around the notch tip was fitted to a large number of nodal displacements obtained from finite element analysis. An over-determined set of simultaneous linear equations was then derived and the nodal displacements reduced to a small set of unknown coefficients by employing the concept of the least-squares method. The efficiency of the proposed method was assessed through analyzing several notched specimens under pure mode I, pure mode II and mixed modes I/II loading. The accuracy of the NSIFs and the coefficients of higher order terms was evaluated by comparing them with the results available in the literature, or the results obtained by the boundary collocation method. While the presented method is simple, it yields very good results.

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1. Introduction

Sharp notches and reentrant corners, especially 90° notches, are found in many engineering structures. Notches may be regarded as sharp when the radius of their tips is very small compared to the length of their sides. Cracks can also be regarded as a particular case of sharp notches in which the notch angle is zero. Welded structures as well as some well-known machine elements such as gears, screws, bolts and nuts are among the practical structures or components that contain V-notches. High local stresses and stress gradients may cause a local damage to notched structures which are subjected to static or variable loads, and greatly affect the load-bearing capacity of the structures. Indeed, the presence of notches severely influences the fatigue and fracture behavior of notched structures. Therefore, many researchers have attempted in the past few decades to develop appropriate approaches for predicting the fracture resistance and the fatigue life of notched components. In order to acquire reliable fatigue and fracture models for engineering structures containing sharp notches, a good understanding of stresses in the vicinity of the notch tip is inevitable. The concept of Linear Elastic Fracture Mechanics (LEFM) is widely used to study the influence of sharp notches on the performance of structures subjected to static or variable loading. The Notch Stress Intensity Factors (NSIFs) characterize the singular stress field around the notch tip and have been extensively employed for predicting not only brittle fracture but also fatigue strength of notched structures. The coefficients of higher order stress terms are also expected to play an important role in the fracture and fatigue processes in notched structures. Hence, an accurate determination of these parameters is of great importance in fracture mechanics.

The linear elastic stress field for a notched plate subjected to an arbitrary in-plane loading which can be expressed in the so-called Williams series expansions [1,2] as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \sum_{n=1}^{\infty} \mathrm{Re}\left\{ \frac{\lambda_n^{\mathrm{I}} A_n}{r^{1-\lambda_n^{\mathrm{I}}}} \begin{Bmatrix} (2+\lambda_n^{\mathrm{I}}\cos 2\alpha+\cos 2\alpha\lambda_n^{\mathrm{I}})\cos(\lambda_n^{\mathrm{I}}-1)\theta-(\lambda_n^{\mathrm{I}}-1)\cos(\lambda_n^{\mathrm{I}}-3)\theta \\ (2-\lambda_n^{\mathrm{I}}\cos 2\alpha-\cos 2\alpha\lambda_n^{\mathrm{I}})\cos(\lambda_n^{\mathrm{I}}-1)\theta+(\lambda_n^{\mathrm{I}}-1)\cos(\lambda_n^{\mathrm{I}}-3)\theta \\ -(\lambda_n^{\mathrm{I}}\cos 2\alpha+\cos 2\alpha\lambda_n^{\mathrm{I}})\sin(\lambda_n^{\mathrm{I}}-1)\theta+(\lambda_n^{\mathrm{I}}-1)\sin(\lambda_n^{\mathrm{I}}-3)\theta \end{Bmatrix} \right\}$$
$$+\sum_{n=1}^{\infty} \mathrm{Re}\left\{ \frac{\lambda_n^{\mathrm{II}} B_n}{r^{1-\lambda_n^{\mathrm{II}}}} \begin{Bmatrix} -(2+\lambda_n^{\mathrm{II}}\cos 2\alpha-\cos 2\alpha\lambda_n^{\mathrm{II}})\sin(\lambda_n^{\mathrm{II}}-1)\theta+(\lambda_n^{\mathrm{II}}-1)\sin(\lambda_n^{\mathrm{II}}-3)\theta \\ (-2+\lambda_n^{\mathrm{II}}\cos 2\alpha-\cos 2\alpha\lambda_n^{\mathrm{II}})\sin(\lambda_n^{\mathrm{II}}-1)\theta-(\lambda_n^{\mathrm{II}}-1)\sin(\lambda_n^{\mathrm{II}}-3)\theta \\ -(\lambda_n^{\mathrm{II}}\cos 2\alpha-\cos 2\alpha\lambda_n^{\mathrm{II}})\cos(\lambda_n^{\mathrm{II}}-1)\theta+(\lambda_n^{\mathrm{II}}-1)\cos(\lambda_n^{\mathrm{II}}-3)\theta \end{Bmatrix} \right\} \quad (1)$$

where r and θ are the polar coordinates as shown in Fig. 1, Re ( ) denotes the real part of ( ), n is the order of term in the infinite series and α is the parameter related to the notch angle (see Fig. 1). λ_n^{I} and λ_n^{II} are the modes I and II eigenvalues obtained from the positive roots of the following characteristic options:

$$\lambda_n^{\mathrm{I}}\sin 2\alpha+\sin 2\lambda_n^{\mathrm{I}}\alpha=0$$
$$\lambda_n^{\mathrm{II}}\sin 2\alpha-\sin 2\lambda_n^{\mathrm{II}}\alpha=0 \quad (2)$$

The terms involving coefficients A_n and B_n ($n \neq 2$ for B_n) also correspond to the modes I and II expansions, respectively. For $n=1$, the coefficients A_1 and B_1 are related to the singular terms, the

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