



# Buckling of symmetrically laminated rectangular plates under parabolic edge compressions

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## ABSTRACT

Buckling analysis of symmetrically laminated rectangular plates with parabolic distributed in-plane compressive loadings along two opposite edges is performed using the Rayleigh–Ritz method. Classical laminated plate theory is adopted. Stress functions satisfying all stress boundary conditions are constructed based on the Chebyshev polynomials. Displacement functions for buckling analysis are constructed by Chebyshev polynomials multiplying with functions that satisfy either simply supported or clamped boundary condition along four edges. Methodology and procedures are worked out in detail. Buckling loads for symmetrically laminated plates with four combinations of boundary conditions are obtained. The proposed method is verified by comparing results to data obtained by the differential quadrature method (DQM) and the finite element method (FEM). Numerical example also shows that the double sine series displacement for simply supported symmetrically laminated plates having bending–twisting coupling may overestimate the stiffness, thus providing higher buckling loads.

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## 1. Introduction

With the increasing usages of composite materials in the field of aerospace, civil, mechanical and marine engineering, the buckling behavior of composite plates subjected to in-plane compressive and/or shear loading has become an important issue to the designers, thus causing much attention. Buckling of isotropic rectangular plates under uniform in-plane compressive loadings has been extensively studied; however, there have been only few previous analyses for the case of nonlinearly distributed in-plane edge loadings. Benoy [1] and van der Neut [2] considered a uniaxial compressive loading with a parabolic distribution and a half sine distribution, respectively. They both used an oversimplified in-plane stress solution for the buckling analysis, in which the  $x$ -direction in-plane normal stress distribution was the same as the applied edge load, and the  $y$ -direction in-plane normal stress and the in-plane shear stress were zero. Bert and Devarakonda [3,4] overcame the deficiencies in the study carried out by Benoy [1] and van der Neut [2], and yielded a more accurate buckling load for the case of a thin rectangular plate under sinusoidal in-plane edge loadings. Jana and Bhaskar [5,6] analyzed the problem of buckling of rectangular plates under non-uniform compressive edge loadings. The in-plane stress components are obtained using a superposition of Airy's stress functions as well as the extended Kantorovich method, buckling loads are then obtained by Galerkin's method. Wang et al. [7–9] obtained buckling load of rectangular plates under nonlinearly distributed edge loadings using the differential quadrature (DQ) method.

It is known that plate problems are often idealizations of portions of a much larger overall stiffened structure, such as an aircraft wing; thus the in-plane compressive edge loads are those exerted by the adjoining free-body on the plate and may not be always uniform. Therefore, buckling of laminated composite plates under non-uniformly distributed edge compressions has always been considered important in view of its great practical relevance [5,6]. The problem of buckling of laminated rectangular plates subjected to uniformly or linearly varying compressive edge loadings has been investigated [10–11]. Similar to the cases of isotropic plates, however, there have been few previous works on the buckling of composite plates under other nonlinearly varying edge loadings. A study on buckling of non-uniformly compressed orthotropic rectangular plates was carried out by Bharat Kalyan and Bhaskar [12]; however, the in-plane stress solution did not satisfy all the stress boundary conditions. Ghomshei and Dameshghie [13] used the DQ method to analyze the buckling of symmetric cross-ply laminated rectangular plates under nonlinearly distributed in-plane edge loadings. Hu et al. [14] investigated the buckling behavior of a symmetrically laminated rectangular plate under parabolic varying in-plane edge loadings; however, they adopted the same oversimplification of the in-plane stress solution as in Benoy [1] and van der Neut [2]. It was pointed out later by Bambill et al. [15] that the buckling loads given by Hu et al. [14] were not correct solutions since the stress functions did not satisfy the compatibility equation. Therefore, the problem in Ref. [14] remains unsolved and deserves further investigations. This is the objective of the present study.

In the present work, buckling behavior of symmetrically laminated rectangular plates under parabolic distributed in-plane edge loadings is investigated. Analytical solutions for buckling load are obtained

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