



# Inelastic buckling of two-layer composite columns with non-linear interface compliance

S. Schnabl\*, I. Planinc

University of Ljubljana, Faculty of Civil and Geodetic Engineering, Jamova 2, SI-1115 Ljubljana, Slovenia

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## ABSTRACT

A mathematical model for a slip-buckling problem has been proposed and its exact solution has been found for the analysis of materially inelastic two-layer composite columns with non-linear interface compliance. The mathematical model has been carried out to evaluate exact critical buckling loads. It has been demonstrated mathematically exactly, that exact critical buckling loads are influenced by the initial stiffness, and hence on linear portion of the interface force-slip relationship. Besides, it has been shown that material inelasticity can reduce the critical buckling loads significantly and that the interlayer stiffness has an important effect on the transition between the elastic and inelastic buckling.

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## 1. Introduction

Layered columns consisting of different or like materials are frequently encountered in a wide range of applications. Due to their high strength-to-weight and stiffness-to-weight ratios, slender composite columns are widely used in aerospace engineering, structural engineering, shipbuilding, and in other branches of industry. In the field of structural engineering typical examples of aforementioned layered structures are steel-concrete and timber-concrete composite columns, layered timber columns, sandwich columns, concrete columns externally reinforced with laminates, and many more. The behavior of these structures largely depends on the type of the connection between the layers and the quality of the used materials.

Since absolutely stiff connection between the layers cannot be achieved in practice, an inter-layer slip between these layers develops, which significantly can affect the mechanical behavior of layered structure. Accordingly, the inter-layer slip has to be taken into consideration in what is called partial interaction analysis of composite structures. Consequently, many published papers that take into account the inter-layer slip analytically or numerically are available in the literature. No attempt is made to discuss it here but the interested reader is referred to the, e.g. [1–14].

The strength of straight layered columns depends to a great extent on their stability and cohesion between the layers. It is

therefore of practical importance to derive exact solutions for such problems. To date only a few exact slip-buckling models of composite columns have been developed; see, for example, [15–20].

In all these previous exact investigations, linear stress-strain relations of the material and linear interfacial constitutive laws between the layers have been assumed. Actually, in reality a highly non-linear material and force-slip law of the interface can be obtained. Notwithstanding, to the best of the authors' knowledge, there exist relatively few exact solutions where linear material an idealized bilinear force-slip law of the interface has been assumed, see, for example [21], but no exact investigation of slip-buckling problem where general inelastic material and non-linear interfacial constitutive law would be employed.

Therefore, the main objective of this paper is to develop a mathematical model for a slip-buckling analysis of geometrically perfect materially inelastic two-layer composite columns with non-linear interfacial compliance between the layers. In many cases, the compliant layer between the layers has been considered to be a very thin interphase layer with vanishing thickness; referred to in the literature as an interface. The interface arises from the damage in composites, e.g. debonding, sliding and cracking across the interface. In this paper, a non-linear tangentially compliant interface layer with non-zero thickness  $d$  is assumed to occupy the area between the layers. The thickness of the interface layer depends on the mechanical properties of the composite, and must be determined experimentally. The mechanical behavior of the interface is described by the general nonuniform, non-linear interface constitutive law.

\* Corresponding author. Tel.: +386 1 47 68 615; fax: +386 1 47 68 629.  
E-mail address: [sschnabl@fgg.uni-lj.si](mailto:sschnabl@fgg.uni-lj.si) (S. Schnabl).