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A new exact analytical approach for free vibration of Reissner–Mindlin functionally graded rectangular plates

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ABSTRACT

An exact closed-form procedure is presented for free vibration analysis of moderately thick rectangular plates having two opposite edges simply supported (i.e. Lévy-type rectangular plates) based on the Reissner–Mindlin plate theory. The material properties change continuously through the thickness of the plate, which can vary according to a power law distribution of the volume fraction of the constituents. By introducing some new potential and auxiliary functions, the displacement fields are analytically obtained for this plate configuration. Several comparison studies with analytical and numerical techniques reported in literature are carried out to establish the high accuracy and reliability of the solutions. Comprehensive benchmark results for natural frequencies of the functionally graded (FG) rectangular plates with six different combinations of boundary conditions (i.e. SSSS–SSSC–SCSC–SCSF–SSSF–SFSF) are tabulated in dimensionless form for various values of aspect ratios, thickness to length ratios and the power law index. Due to the inherent features of the present exact closed-form solution, the present results will be a useful benchmark for evaluating the accuracy of other analytical and numerical methods, which will be developed by researchers in the future.

1. Introduction

Functionally graded materials (FGMs) are a new class of composite structures that is of great interest for engineering design and manufacture. These kinds of materials possess desirable properties for specific applications, particularly for aircrafts, space vehicles, optical, biomechanical, electronic, chemical, mechanical, shipbuilding and other engineering structures under stress concentration, high thermal and residual stresses. FGMs are heterogeneous composite materials, in which the material properties vary continuously from one interface to the other. This is achieved by gradually varying the volume fraction of the constituent materials.

FG plate problems deal with two main concepts: the modeling of the plate (plate theories) and the procedure of solution. Most commonly used plate theories can be classified into three main categories: thin plate theory [1] (e.g. Kirchhoff theory or CPT), moderately thick plate theory [2] (e.g. first-order shear deformation plate theory of Mindlin or FSDT) and thick plate theory [3–5] (e.g. third-order shear deformation plate theory of Reddy or TSDT, higher-order shear deformation plate theory or HSDT and three-dimensional (3-D) elasticity theory). The governing equation of aforementioned plate theories must be solved through three types of solution: numerical methods [6–20] (e.g., Ritz energy method, finite element method, differential quadrature method, Galerkin method), semi-analytical methods [21–29] (e.g. power series method) or exact analytical methods [30–32] (e.g. state-space method). It is true that all researchers would like to present exact solutions for their FG plate problems on the basis of 3-D elasticity solution in which no assumptions are made. However, because of the mathematical and computational complexities, exact solutions for vibratory characteristics of FG plates are available only for simple cases (i.e., an elastic plate with either simply supported boundary conditions based on the 3-D elasticity theory or different boundary conditions based on simplified theories such as the CPT).

According to a comprehensive survey of literature, it is found that a wide range of researches has been carried out on free vibration of the FG plates that the most of them used numerical solution methods. Free vibration of FG simply supported and clamped rectangular thin plates was considered by Abrate [6] using the CPT. Also, Abrate [7] analyzed free vibration, buckling and static deflections of FG square, circular and skew plates with different combinations of boundary conditions on the basis of the CPT, FSDT and TSDT. Qian et al. [8,9] carried out an analysis of free and forced vibrations of both homogeneous and FG thick plates with the higher-order shear and normal deformable plate theory using meshless local Petrov–Galerkin method. Free vibration analysis of FG simply supported square plates was studied by Pradyumna and Bandyopadhyay [10] using a higher order finite

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