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Short communication

A two-parameter model of the effective elastic tensor for cortical bone

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ABSTRACT

Multiscale models of cortical bone elasticity require a large number of parameters to describe the organization and composition of the tissue. We hypothesize that the macro-scale anisotropic elastic properties of different bones can be modeled retaining only two variable parameters, and setting the others to universal values identical for all bones. Cortical bone is regarded as a two-phase composite material: a dense mineralized matrix (ultrastructure) and a soft phase (pores). The ultrastructure is assumed to be a homogeneous and transversely isotropic tissue whose elastic properties in different directions are mutually dependent and can be scaled with a single parameter driving the overall rigidity. This parameter is taken to be the volume fraction of mineral f_{ha}. The pore network is modeled as an ensemble of water-filled cylinders and described only by the porosity p. The effective macroscopic elasticity tensor $C_{ij}(f_{ha},p)$ is calculated with a multiscale micromechanics approach starting from existing models. The modeled stiffness coefficients compare favorably to four literature datasets which were chosen because they provide the full stiffness tensors of groups of human samples. Since the physical counterparts of f_{ha} and p were unknown for the datasets, their values which allow the best fit of experimental tensors by the modeled ones were determined by optimization. Optimum values of f_{ha} and p are found to be unique and realistic. These results suggest that a two-parameter model may be sufficient to model the elasticity of different samples of human femora and tibiae. Such a model would in particular be useful in large-scale parametric studies of bone mechanical response.

1. Introduction

Macroscopic effective elastic coefficients of cortical bone can be predicted by multiscale models given relative amounts of the elementary constituents and assumed organizational patterns. Multiscale models generally require a large number of parameters to describe the tissue at each scale. For instance, authors have considered variations of the volume fraction of vascular porosity (Hellmich et al., 2004b; Baron et al., 2007; Parnell and Grimal, 2009); volume fraction of pores at different scales (Sevostianov and Kachanov, 2000); relative area of osteonal, interstitial tissue and resorption cavities (Dong and Guo, 2006); organization patterns of the mineral (Crolet et al., 2005); patterns of osteonal lamellae (Crolet et al., 1993; Aoubiza et al., 1996). In addition, the models usually consider different volume fractions of elementary constituents. Models with a large number of parameters are difficult to validate since several combinations of the parameter values may result in similar effective elasticity values to be compared to experiments.

This communication investigates a simple multiscale model of mature Haversian bone elasticity. We hypothesize that macro-scale anisotropic properties of different bones can be recovered to some extent by a model using only two parameters: porosity and mineral content. Elastic constants of the phases and organizational patterns are fixed for all bones. The model lends itself to a comprehensive analysis of the role of each parameter and can be critically assessed with experimental data.

2. Method

Cortical bone is regarded as a two-phase composite with a transversely isotropic (TI) mineralized matrix (ultrastructure) pervaded by cylindrical pores (vascular porosity). The 'mineral foam matrix with collagen inclusions' micromechanical model (Hellmich et al., 2004a) is used for the ultrastructure because of its limited number of variables. It is based on two idealizations: (1) at a length scale of 100 nm, hydroxyapatite crystals and ultrastructural water with non-collagenous organic material constitutes a mineral foam; (2) at a length scale of 5 to 10 microns, collagen fibers are embedded into the mineral foam. The stiffness

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