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# Assembling mode shapes by least squares

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#### ABSTRACT

Assembling (or 'gluing') mode shapes identified from multiple setups is a problem frequently encountered in full-scale modal tests that cover a large number of locations with a limited number of sensors. Mode shapes identified in individual setups can have different sense and scaling. Depending on the number of reference degrees-of-freedom (dofs) and the quality of identified mode shapes, implementation issues can arise when determining the optimal mode shape that compromise among different setups. This paper presents a theory with an automated procedure for determining the optimal mode shape that fits the mode shapes identified from multiple setups in a least square sense. The measure-of-fit function is defined as the squared difference between the theoretical and identified mode shapes suitably oriented and scaled to the same norm. Due to the nonlinear nature of the objective function, the optimal mode shape cannot be determined analytically as in conventional least square problems. A fast iterative procedure is proposed, making use of partially optimal solutions that can be derived analytically. The proposed method can be implemented in an automated manner without the need to select the reference dof or setup for scaling purpose. It is applied to assembling mode shapes identified from ambient vibration tests of two full-scale structures.

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### 1. Introduction

Modal identification involves the determination of natural frequencies, damping ratios and mode shapes from measured vibration data of the structure in its constructed state [1,2]. Attributed to advances in sensing technology and identification methods (e.g., [3–8]), full-scale tests have become a feasible option for assessing the actual dynamic properties of structures ([4,9–11]). Besides natural frequency and damping ratios, mode shapes identified from vibration tests provide insights into the nature of modes through relative motions at the measured degrees-of-freedom (dofs) of the structure. While modern sensing and data acquisition technology have allowed high resolution time history data to be obtained, the spatial resolution of mode shape can only be improved by increasing the number of sensing locations. Due to limited instrumentation budget and practical difficulties in deploying a large array of sensors with synchronous data acquisition, full mode shapes with a large number of measured dofs are often assembled from partial (local) ones identified from individual setups, each covering a different part of the structure. Since the mode shapes identified from individual setups are arbitrary in sense and scaling, common 'reference' dofs must be present across different setups in order to allow their mode shapes to be assembled. The reference dofs must have significant frequency response in the modes of interest. There must also be at least one common reference dof across any two setups.

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