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ScienceDirect
Mathematics and Computers in Simulation 82 (2012) 1185–1218
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Numerical study of the KP equation for non-periodic waves

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Received 7 February 2010; received in revised form 25 April 2010; accepted 19 May 2010
Available online 11 June 2010

Abstract

The Kadomtsev–Petviashvili (KP) equation describes weakly dispersive and small amplitude waves propagating in a quasi-two-dimensional situation. Recently a large variety of exact soliton solutions of the KP equation has been found and classified. Those soliton solutions are localized along certain lines in a two-dimensional plane and decay exponentially everywhere else, and they are called line-soliton solutions in this paper. The classification is based on the far-field patterns of the solutions which consist of a finite number of line-solitons. In this paper, we study the initial value problem of the KP equation with V- and X-shape initial waves consisting of two distinct line-solitons by means of the direct numerical simulation. We then show that the solution converges asymptotically to some of those exact soliton solutions. The convergence is in a locally defined L^2-sense. The initial wave patterns considered in this paper are related to the rogue waves generated by nonlinear wave interactions in shallow water wave problem.

Kadomtsev–Petviashvili equation; Soliton solutions; Chord diagrams; Pseudo-spectral method; Window technique

1. Introduction

The KdV equation may be obtained in the leading order approximation of an asymptotic perturbation theory for one-dimensional nonlinear waves under the assumptions of weak nonlinearity (small amplitude) and weak dispersion (long waves). The initial value problem of the KdV equation has been extensively studied by means of the method of inverse scattering transform (IST). It is then well-known that a general initial data decaying rapidly for large spatial variable evolves into a sum of individual solitons and some weak dispersive wave trains separate away from solitons (see for examples [1,17,19,24,5]).

In 1970, Kadomtsev and Petviashvili [9] proposed a two-dimensional dispersive wave equation to study the stability of one soliton solution of the KdV equation under the influence of weak transversal perturbations. This equation is now referred to as the KP equation, and considered to be a prototype of the integrable nonlinear dispersive wave equations in two dimensions. The KP equation can be also represented in the Lax form, that is, there exists a pair of linear equations associated with the eigenvalue problem and the evolution of the eigenfunctions. However, unlike the case of the KdV equation, the method of IST based on the pair of linear equations does not seem to provide a practical method for the initial value problem with non-periodic waves considered in this paper. At the present time, there is no feasible analytic method to solve the initial value problem of the KP equation with initial waves having line-solitons in the far field.

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0378-4754/$36.00
doi:10.1016/j.matcom.2010.05.025