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Original article

Development of a meshless Galerkin boundary node method for viscous fluid flows

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Abstract

In this paper, a meshless Galerkin boundary node method is developed for boundary-only analysis of the interior and exterior incompressible viscous fluid flows, governed by the Stokes equations, in biharmonic stream function formulation. This method combines scattered points and boundary integral equations. Some of the novel features of this meshless scheme are boundary conditions can be enforced directly and easily despite the meshless shape functions lack the delta function property, and system matrices are symmetric and positive definite. The error analysis and convergence study of both velocity and pressure are presented in Sobolev spaces. The performance of this approach is illustrated and assessed through some numerical examples.

Keywords: Meshless; Galerkin boundary node method; Boundary integral equations; Stokes equations; Stream function

1. Introduction

Let Ω be a bounded domain of $\mathbb{R}^2$ with boundary Γ and let Ω' denote the complement of $\overline{\Omega} = \Omega \cup \Gamma$, that is the exterior of $\overline{\Omega}$. A point of $\mathbb{R}^2$ is denoted by $\mathbf{x} = (x_1, x_2)$ or $\mathbf{y} = (y_1, y_2)$. In this paper, we are concerned with a boundary-type meshless method for the following interior and exterior steady-state Stokes problem which describes the motion of a slow and viscous incompressible fluid

$$\begin{cases} -\nu\,\Delta\mathbf{u} + \nabla p = 0, & \text{in } \Omega \text{ or } \Omega', \\ \nabla\cdot\mathbf{u} = 0, & \text{in } \Omega \text{ or } \Omega', \\ \mathbf{u} = \mathbf{g}, & \text{on } \Gamma, \\ |u_j(\mathbf{x})| = O(1), & j = 1, 2, \text{ as } |\mathbf{x}| \to \infty, \end{cases} \tag{1}$$

where the symbols Δ,∇ and $\nabla\cdot$ stand for the Laplacian, gradient and divergence operators, respectively; $\mathbf{u} = (u_1, u_2)$ is the velocity vector; p is the pressure; ν is the given constant viscosity of the fluid; $\mathrm{g} \in (H_0^{1/2}(\Gamma))^2 := \{\mathrm{f} : \mathrm{f} \in$

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