

Original article

Dynamics of an ecological model with impulsive control strategy and distributed time delay

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Abstract

In this paper, using the theories and methods of ecology and ordinary differential equations, an ecological model with an impulsive control strategy and a distributed time delay is defined. Using the theory of the impulsive equation, small-amplitude perturbations, and comparative techniques, a condition is identified which guarantees the global asymptotic stability of the prey-(x) and predator-(y) eradication periodic solution. It is proved that the system is permanent. Furthermore, the influences of impulsive perturbations on the inherent oscillation are studied numerically, an oscillation which exhibits rich dynamics including period-halving bifurcation, chaotic narrow or wide windows, and chaotic crises. Computation of the largest Lyapunov exponent confirms the chaotic dynamic behavior of the model. All these results may be useful for study of the dynamic complexity of ecosystems.

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1. Introduction

The dynamical relationships between predators and their preys have been and will continue to be one of the dominant themes in both ecology and mathematical ecology because of its universal existence and importance [15]. At present it is clear that predator–prey functional responses have ability to influence the dynamical behaviors of predator–prey systems, especially predator–prey systems with some impulse perturbations, all these results are exhibited in these papers [2,1,30,37]. Many evolutionary processes are characterized by the fact that at certain moments they experience an abrupt change of state. These processes are subject to short-term perturbations of negligible duration compared with the duration of the process [13,14]. Consequently, it is natural to assume that these perturbations act instantaneously, that is, in the form of an impulse. It is well known that many biological phenomena involving thresholds, such as bursting rhythm models in medicine and biology, optimal control models in economics and pharmacokinetics, and frequency-modulated systems exhibit impulse effects. Thus impulsive differential equations, or differential equations involving impulsive effects, appear to be a natural description of observed evolution phenomena for several real-world problems [18,3].

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