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Spatial stress and strain distributions of viscoelastic layers in oscillatory shear

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Abstract

One of the standard experimental probes of a viscoelastic material is to measure the response of a layer trapped between parallel surfaces, imposing either periodic stress or strain at one boundary and measuring the other. The relative phase between stress and strain yields solid-like and liquid-like properties, called the storage and loss moduli, respectively, which are then captured over a range of imposed frequencies. Rarely are the full spatial distributions of shear and normal stresses considered, primarily because they cannot be measured except at boundaries and the information was not deemed of particular interest in theoretical studies. Likewise, strain distributions throughout the layer were traditionally ignored except in a classical protocol of Ferry, Adler and Sawyer, based on snapshots of standing shear waves. Recent investigations of thin lung mucus layers exposed to oscillatory stress (breathing) and strain (coordinated cilia), however, suggest that the wide range of healthy conditions and environmental or disease assaults lead to conditions that are quite disparate from the "surface loading" and "gap loading" conditions that characterize classical rheometers. In this article, we extend our previous linear and nonlinear models of boundary stresses in controlled oscillatory strain to the entire layer. To illustrate non-intuitive heterogeneous responses, we characterize experimental conditions and material parameter ranges where the maximum stresses migrate into the channel interior.

Keywords: Rheology; Viscoelastic; Oscillatory shear; Upper convected Maxwell; Giesekus

1. Introduction

We first recall the formulation developed in [7,11,12], which is a generalization of the Ferry shear wave model [4–6] (also considered by Schrag et al. [15,16] for linear viscoelastic experiments) to finite depth layers and nonlinear constitutive laws. We briefly summarize the key elements from these references in order to describe the present focus

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