Available online at www.sciencedirect.com

ScienceDirect

Mathematics and Computers in Simulation 82 (2012) 946–957

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Original Articles

Excited Bose–Einstein condensates: Quadrupole oscillations and dark solitons

V.A. Achilleos [a], T.P. Horikis [b], G. Theocharis [c,*], P.G. Kevrekidis [c], D.J. Frantzeskakis [a]

[a] Department of Physics, University of Athens, Panepistimiopolis, Zografos, Athens 15784, Greece
[b] Department of Mathematics, University of Ioannina, Ioannina 45110, Greece
[c] Department of Mathematics and Statistics, University of Massachusetts, Amherst, MA 01003-4515, USA

Received 25 November 2009; received in revised form 17 July 2010; accepted 21 July 2010
Available online 30 July 2010

Abstract

We study the dynamics of atomic Bose–Einstein condensates (BECs), when the quadrupole mode is excited. Within the Thomas–Fermi approximation, we derive an exact first-order system of differential equations that describes the parameters of the BEC wave function. Using perturbation theory arguments, we derive explicit analytical expressions for the phase, density and width of the condensate. Furthermore, it is found that the observed oscillatory dynamics of the BEC density can even reach a quasi-resonance state when the trap strength varies according to a time-periodic driving term. Finally, the dynamics of a dark soliton on top of a breathing BEC are also briefly discussed.

PACS: 03.75.Kk; 03.75.Lm; 05.45.Yv

Keywords: Bose–Einstein condensates; Quadrupole oscillations; Dark solitons; Perturbation theory

1. Introduction

The realization of Bose–Einstein condensates (BECs) in dilute atomic vapors, which was awarded the Nobel prize in physics in 2001 [9,15], has been a fundamental development in quantum and atomic physics in the last decades. The statics and dynamics of BECs can be described by means of an effective mean-field model, known as the Gross–Pitaevskii (GP) equation [10], which is a variant of the well-known nonlinear Schrödinger (NLS) equation. This description allows for an understanding of fundamental properties of BECs, including their *collective excitations* [10]. These refer to the internal oscillatory modes of the system and confirm the superfluid nature of the condensate. They are usually studied in the framework of the hydrodynamic approach for BECs, but also in the context of the ensuing Bogoliubov–de Gennes (BdG) equations when linearizing around its stationary states [10]. Importantly, collective oscillations have also been studied experimentally, with a pertinent example being the so-called *quadrupole* oscillation [20,27]. This oscillation was observed in an axisymmetric cigar-shaped BEC, which executed shape oscillations due to a temporal variation of the trap frequency. The value of the characteristic frequency of the quadrupole oscillation found in the experiment was in excellent agreement with the one predicted theoretically [1,23,28].

* Corresponding author.
E-mail address: georgiostheocharis@gmail.com (G. Theocharis).

0378-4754/$36.00
doi:10.1016/j.matcom.2010.07.019