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Original article

Numerical implementation of the asymptotic boundary conditions for steadily propagating 2D solitons of Boussinesq type equations

Christo I. Christov *

Department of Mathematics, University of Louisiana at Lafayette, P.O. Box 1040, Lafeyette, LA 70504-1010, USA

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Abstract

In the present paper, a difference scheme on a non-uniform grid is constructed for the stationary propagating localized waves of the 2D Boussinesq equation in an infinite region. Using an argument stemming form a perturbation expansion for small wave phase speeds, the asymptotic decay of the wave profile is identified as second-order algebraic. For algebraically decaying solution a new kind of nonlocal boundary condition is derived, which allows to rigorously project the asymptotic boundary condition at the boundary of a finite-size computational box. The difference approximation of this condition together with the bifurcation condition complete the algorithm. Numerous numerical validations are performed and it is shown that the results comply with the second-order estimate for the truncation error even at the boundary lines of the grid. Results are obtained for different values of the so-called 'rotational inertia' and for different subcritical phase speeds. It is found that the limits of existence of the 2D solution roughly correspond to the similar limits on the phase speed that ensure the existence of subcritical 1D stationary propagating waves of the Boussinesq equation.

Keywords: Two-dimensional Boussinesq equation; Difference scheme; Asymptotic boundary condition; Non-uniform grid

1. Introduction

Boussinesq's equation (BE) was the first model for surface waves in shallow fluid layer that accounts for both nonlinearity and dispersion. The balance between the steepening effect of the nonlinearity and the flattening effect of the dispersion maintains the shape of the wave. The above described balance is a new paradigm in physics and can be properly termed 'Boussinesq Paradigm'. In a coordinate frame moving with the center of the propagating wave, BE reduces to Korteweg-de Vries which is widely studied in 1D.

One of the most important features of the generalized wave equations containing nonlinearity and dispersion, is that they possess solutions of type of permanent waves as shown in the original Boussinsq work [4]. In the 1960s it was discovered that these permanent waves can behave in many instances as particles (the so-called 'collision property'), and were called *solitons* by Zabusky and Kruskal [24]. Currently the coinage 'soliton' is reserved only for the particle-like waves that are solutions of a fully integrable model, such as the original Boussinesq equation. The localized waves which can retain their identity during interaction appear to be a rather pertinent model for particles, especially

* Tel.: +1 337 482 5273; fax: +1 337 482 5346.
E-mail address: christov@louisiana.edu.

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