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A consistent stable numerical scheme for a nonlinear option pricing model in illiquid markets

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Abstract

Markets liquidity is an issue of very high concern in financial risk management. In a perfect liquid market the option pricing model becomes the well-known linear Black–Scholes problem. Nonlinear models appear when transaction costs or illiquid market effects are taken into account. This paper deals with the numerical analysis of nonlinear Black–Scholes equations modeling illiquid markets when price impact in the underlying asset market affects the replication of a European contingent claim. Numerical analysis of a nonlinear model is necessary because disregarded computations may waste a good mathematical model. In this paper we propose a finite-difference numerical scheme that guarantees positivity of the solution as well as stability and consistency.

Nonlinear Numerical Analysis; Simulation; Option Pricing; Illiquid Markets

1. Introduction

An option is a financial contract entered into by two parties, a buyer and a seller. The buyer of the contract obtains the right to trade an underlying asset, such as a stock, for a specified price, called the strike, on or before a maturity date. Options providing the right to buy the underlying asset are known as calls, whereas options conferring the right to sell the underlying asset are referred to as puts.

There are many varieties of options. European options may only be exercised on the maturity date. American options may be exercised any time up to and including the maturity date, see Ref. [12] for details.

One of the modern financial theory's biggest successes in terms of both approach and applicability has been Black–Scholes pricing, which allows investors to calculate the 'fair' price of a derivative security whose value depends on the value of another security, known as the underlying, based on a small set of assumptions on the price behaviour of that underlying. One of the major assumptions of Black–Scholes model is that the market in the underlying asset is perfectly elastic so that large trades do not affect prices in equilibrium. This occurs in perfectly liquid markets, but the case is clearly unrealistic.

The presence of price impact of investors' trading has been widely documented and extensively analyzed in the literature, see, for instance, [22,13]. In the presence of asymmetric information, Refs. [15,2] use an equilibrium approach to investigate how informed traders reveal information and affect the market price through trading. [25]

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