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Original article

A particle method for a collisionless plasma with infinite mass

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Abstract

The one-dimensional Vlasov–Poisson system is considered and a particle method is developed to approximate solutions without compact support which tend to a fixed background of charge as $|x| \to \infty$. Such a system of equations can be used to model kinetic phenomena occurring in plasma physics. A localized particle method is constructed and implemented using the fact that solutions to the Vlasov–Poisson system propagate at finite speeds. Finally, the numerical method is utilized to ascertain information regarding the time asymptotics of the generated electrostatic field.

Keywords: Plasma; Particle method; Vlasov–Poisson; Infinite mass

1. Introduction

The motion of a collisionless plasma – an ionized gas of high-temperature and low-density – is described by the Vlasov–Maxwell system. If this fundamental kinetic model is posed in a two-dimensional phase space (one for spatial variables and another representing momentum) then Maxwell's equations simplify greatly and the problem reduces to the one-dimensional Vlasov–Poisson system. We consider this system of PDEs for the motion of negative charges upon prescribing a fixed, spatially-homogeneous background of positive ions:

$$\left.\begin{aligned} \partial_t f + v\partial_x f - E\partial_v f &= 0, \\ \partial_x E(t,x) &= \int (F(v) - f(t,x,v))dv \\ f(0,x,v) &= f_0(x,v). \end{aligned}\right\} \tag{1}$$

Here, $t \geq 0$ denotes time, $x \in \mathbb{R}$ is position, $v \in \mathbb{R}$ is momentum, $f(t, x, v)$ represents the density of negative ions or electrons, $F(v) \not\equiv 0$ is a given function describing the fixed background of positive charge, and $E(t, x)$ represents the electric field generated by the charges. We define

$$\rho(t,x) := \int (F(v) - f(t,x,v))dv$$

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