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Original article

Application of a modified semismooth Newton method to some elasto-plastic problems

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Abstract

Some elasto-plasticity models with hardening are discussed and some incremental finite element methods with different time discretisation schemes are considered. The corresponding one-time-step problems lead to variational equations with various non-linear operators. Common properties of the non-linear operators are derived and consequently a general problem is formulated. The problem can be solved by Newton-like methods. First, the semismooth Newton method is analysed. The local superlinear convergence is proved in dependence on the finite element discretisation parameter. Then it is introduced a modified semismooth Newton method which contain suitable "damping" in each Newton iteration in addition. The determination of the damping coefficients uses the fact that the investigated problem can be formulated as a minimisation one. The method is globally convergent, independently on the discretisation parameter. Moreover the local superlinear convergence also holds. The influence of inexact inner solvers is also discussed. The method is illustrated on a numerical example.

Keywords: Elasto-plasticity; Hardening; Incremental finite element method; Semismooth Newton method; Damping

1. Introduction

Elasto-plastic problems are usually formulated as quasi-static initial boundary value problems, so the history of loading is taken into account. Elasto-plasticity has been investigated for example in [3,4,11,13,14,17,10] from a mathematical point of view and in [8] from an engineering point of view. We will deal with associative plasticity of von Mises' type with hardening and we will focus on time-discretised elasto-plastic problems.

Time discretisation of constitutive elasto-plastic problems can be based on the explicit and implicit Euler methods or the return mapping concept, see [4,8]. By time discretisation, we obtain a non-linear stress–strain relation, eventually the corresponding relation between stress and strain increments, which is considered here. The form of the relation depends on a chosen time discretisation scheme, yield functions or plastic flow and hardening rules. On the other hand, the stress–strain operators have many common properties typical for associative plasticity of von Mises type with hardening. Therefore we will consider a general operator with suitable assumptions.

If we substitute the stress–strain relation into the equilibrium equation, we can formulate the one-time-step elasto-plastic problem by non-linear variational equation with respect to unknown displacement fields, see e.g. [3,4,14,10].

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