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Original article

Algorithmic detection of hypercircles

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Abstract

In the algebraically optimal reparametrization problem, one of the possible approaches deals with computing a parametric variety of Weyl and checking whether this variety is a hypercircle. Here, algorithms to detect whether a curve given parametrically is a hypercircle are provided.

Keywords: Rational curves; Rational parametrizations; Field of definition; Simplification of parametrizations

1. Introduction

We can think of the real plane as the field of complex numbers $\mathbb{C}$, an algebraic extension of the reals $\mathbb{R}$ of degree 2. Analogously, we can consider a characteristic zero base field $\mathbb{K}$ and an algebraic extension of degree d, $\mathbb{K}(\alpha)$. Since elements in $\mathbb{K}(\alpha)$ can be expressed uniquely as $a_0+a_1\alpha+\cdots+a_{d-1}\alpha^{d-1}$, with $a_i\in\mathbb{K}$, $\mathbb{K}(\alpha)$ can be identified with the vector space $\mathbb{K}^d$, via the base $\{1, \alpha, \ldots, \alpha^{d-1}\}$.

Then, recall that a real circle can be defined as the image (in the real plane, suitably identified with the complex numbers) of the real axis under a Moebius transformation in the complex field. Likewise, and roughly speaking, a *hypercircle* (i.e. a kind of non-standard circle) can be defined as the curve in $\mathbb{K}^d$ that is the image of "the $\mathbb{K}$-axis" under the transformation $(\mathfrak{a}t+\mathfrak{b}/\mathfrak{c}t+\mathfrak{d}):\mathbb{K}(\alpha)\to\mathbb{K}(\alpha)$ where $\mathfrak{a}\mathfrak{d}-\mathfrak{b}\mathfrak{c}\neq 0$; we will see later, in Definition 2, that we consider in fact the hypercircle in the d-dimensional affine space over the algebraic closure of $\mathbb{K}$ instead of over $\mathbb{K}$. This type of curves has been introduced in [1] and studied in detail in [4].

For example, if we take $\mathbb{K}=\mathbb{Q}$, let α be such that $\alpha^3+2=0$, and finally the map $u(t)=(t+\alpha/t-\alpha)$, then $u(t)$ can be written uniquely as $\phi_0(t)+\alpha\phi_1(t)+\alpha^2\phi_2(t)$, where $\phi_i\in\mathbb{K}(t)$, as $u(t)=(t^3-2/2+t^3)+\alpha(2t^2/2+t^3)+\alpha^2(2t/2+t^3)$. The rational functions $\phi_i(t)$ define the following hypercircle in three-dimensional space:

$$\left(\frac{t^3-2}{2+t^3}, \frac{2t^2}{2+t^3}, \frac{2t}{2+t^3}\right)$$

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