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Original article

Algebraic combinatorics of diametric magic circles

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Abstract

We provide a definition of a diametric magic circle of order n. In this paper, we use techniques in computational algebraic combinatorics and enumerative geometry to construct and to count d-magic circles. We also provide a description of its minimal Hilbert basis and determine the symmetry operations on d-magic circles. Finally, we give an algorithm for writing a natural d-magic circle in terms of the minimal Hilbert basis.

MSC: 05A15; 15A48

Keywords: Magic circles; Enumeration; Polyhedral cones; Ehrhart series; Minimal Hilbert basis

1. Introduction

Magic squares, magic circles, and other *magic* arrangements of nonnegative integers that satisfy certain summation properties have occupied a popular part of the recreational mathematics landscape for millennia. A *semi-magic square* is an arrangement of whole numbers in a square grid such that the row and column sums are all equal. If, in addition, both main diagonals share this sum, the arrangement is called a magic square. The earliest known magic square dates back to the Lo Shu (River Plan, c. 2700 B.C.) possibly referring to the magic square of order 3. Numerous articles and books have been written in analyzing and constructing these objects, from ancient manuscripts to modern doctoral dissertations [4,12,2].

On the other hand, a magic circle is an arrangement of numbers in a circular grid which satisfies certain summation properties. In the mid 1700s, the American founding father Dr. Benjamin Franklin constructed a magic circle with many astonishing attributes, but most familiarly, whose *annular* sums and *radial* sums were equal. We call this type of magic circle a radial magic circle, or *r-magic circle*.

There is an obvious correspondence between r-magic circles and semi-magic squares. By fixing an ordering on the annuli and radii of an r-magic circle, one can construct a semi-magic square by carefully writing each annulus as a row in a square grid, as in Fig. 1.

Thus, in this general setting, the study of r-magic circles boils down to the study of semi-magic squares. As an aside, in the case of Benjamin Franklin's r-magic circle, it turns out that this obvious correspondence does not, however, preserve all the other astonishing attributes of Franklin's magic circle. More on Franklin's magic circle can be found in [8].

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