

Original article

From general state-space to VARMAX models

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Abstract

We propose two new algorithms to go from any state-space model to an output equivalent and invertible Vector Autoregressive Moving Average model with exogenous regressors (VARMAX). As the literature shows how to do the inverse transformation, these results imply that both representations, state-space and VARMAX, are equally general and freely interchangeable. These algorithms are useful to solve three practical problems: (i) discussing the identifiability of a state-space model, (ii) performing its diagnostic checking, and (iii) calibrating its parameters so that it realizes, exactly or approximately, a given reduced-form VARMAX. These applications are illustrated by means of practical examples with real data.

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1. Introduction

It is well known that any VARMAX process can be written in an equivalent state-space (SS) form [see, e.g., 1]. Then, it is natural to ask whether it is possible to perform the inverse transformation, i.e., to derive the coefficients of the VARMAX model observationally equivalent to a given SS representation. Our results provide an affirmative answer to this question and, as a consequence, choosing between both representations is just a matter of convenience.

In this framework, the contribution of this paper is twofold.

First, it provides the formal grounds for the answer given above by describing two algorithms to compute the coefficients of a VARMAX model, in its *standard* [14] or *echelon* [6] form, equivalent to a general fixed-coefficients SS model. The first procedure is simpler, but requires every component of the endogenous variable to have the same dynamic order, i.e., their so-called observability or Kronecker indices must be equal. The second method, which refines in several ways the results of [5], is more complex but does not constrain the model dynamics. With a minor variation, the same procedures can also be applied to time-varying parameter models.

On the basis of these algorithms, our second contribution consists of discussing their practical use in three areas: (i) analyzing the identifiability of a SS model, (ii) performing its diagnostic checking, and (iii) calibrating its parameters so that it realizes, exactly or approximately, a given reduced-form VARMAX.

The structure of the paper is as follows. Section 2 presents the VARMAX and SS representations that will be used in the rest of the article and summarizes some previous results. Section 3 describes the algorithms proposed and details how to implement them in practice. Section 4 provides some examples illustrating all the practical applications

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