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Original article

Simulation of the CEV process and the local martingale property[☆]

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Abstract

We consider the constant elasticity of variance (CEV) process, reviewing the relationships between its transition density and that of the non-central chi-squared distribution. When the CEV parameter exceeds one, the forward price process is a strictly local martingale, and the price of a plain vanilla European call option reflects this fact. We develop techniques for Monte Carlo simulation of the CEV process, for all parameter regimes, and compare the results against the analytic expressions for plain vanilla European option prices. Using these techniques, we also verify the local martingale property.

Keywords: CEV process; Bessel process; Local martingale

1. Introduction

Pricing derivatives under the assumption of constant volatility, as in the classic Black–Scholes–Merton model [3,19] of option pricing, is well-known to give results which cannot be reconciled with market observations, although these problems did not widely manifest themselves until the 1987 market crash. After this event, the so-called *volatility smile* or *volatility skew* became common place in equity markets.

The volatility smile is a market phenomenon whereby the Black–Scholes implied volatility of an option exhibits a dependence on the strike price. An alternative to the Black–Scholes model, which exhibits such a volatility skew, is the constant elasticity of variance (CEV) process, first proposed by Cox and Ross [5].

The CEV model is a continuous time diffusion process satisfying[2]

$$\mathrm{d}F = \sigma F^{\alpha}\,\mathrm{d}W, \quad F(0) = F_0 > 0, \tag{1}$$

where $F(t)$ is the state variable representing the forward price of some underlying asset at time t and W is a standard Brownian motion. The parameter α is called the elasticity and we take $\alpha \neq 1$ to distinguish (1) from the Black–Scholes model. To ensure σ has the correct dimensions, we take $\sigma = \sigma_{\mathrm{LN}} F_0^{1-\alpha}$, where σ_{LN} is the effective lognormal volatility.

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2 We ignore a possible drift term here, but its inclusion is straightforward, and has no substantial effects.

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