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Computational algebraic algorithms for the reliability of generalized *k*-out-of-*n* and related systems

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Abstract

Identities and bounds for the reliability of coherent systems are analysed and computed using the techniques of commutative algebra. The techniques are applied to the analysis of some of the most relevant *k*-out-of-*n* class systems. The efficiency of the algebraic approach in obtaining exact identities, bounds and asymptotic formulas shows good performance when compared with results from the literature. The papers points to some new applications of these techniques that emphasize the connection of algebra and probability in this context.

Keywords: System reliabilty; *k*-out-of-*n* Systems; Monomial ideals; Minimal free resolutions

1. Introduction

Every multi-state coherent system, such as a network, has a monomial ideal associated with it [12]. The knowledge of some important numerical invariants of this ideal provides exact reliability identities and good reliability bounds for the corresponding system. However, the computation of the necessary invariants, in particular the multigraded Hilbert series and Betti numbers of large monomial ideals is a computationally hard task. The techniques and algorithms developed by the authors [19,20] allow efficient computations and a close analysis of some of the most relevant systems in reliability theory. Actual applications of coherent systems to problems on networks, project management, etc., are stressed showing that the algebraic analysis and algorithms yield an efficient way to evaluate their reliability. For a range of other problems, when there are no combinatorial formulas or asymptotics results the computer algebra algorithms still give fast accurate solutions for relatively large systems.

An important family of coherent systems which has been the object of considerable attention in the reliability literature is that of *k*-out-of-*n* systems, introduced in [1]. The most relevant *k*-out-of-*n* systems include consecutive *k*-out-of-*n* systems [15], which have also been at the centre of the study of scan statistics, used, for example, in gene association studies. Other relevant *k*-out-of-*n* systems are weighted *k*-out-of-*n* [22], which are used to benchmark the methods in this paper.

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