

Original article

Ill-posedness of nonlinear parabolic equation with critical initial condition

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Abstract

We study the initial boundary value problem of strongly nonlinear parabolic equation:

$$\frac{\partial u}{\partial t} = \sum_{i=1}^N \frac{\partial}{\partial x_i} \left(\left| \frac{\partial u}{\partial x_i} \right|^{p-2} \frac{\partial u}{\partial x_i} \right) + u^{1+\alpha}$$

with critical initial data. By introducing a family of potential wells the global nonexistence of solution is proved, which solves the open problem existing in some classical literature.

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Keywords: Nonlinear parabolic equation; Critical initial data; Potential wells; Global nonexistence

1. Introduction

We consider the initial boundary value problem of nonlinear parabolic equation:

$$\frac{\partial u}{\partial t} = \sum_{i=1}^n \frac{\partial}{\partial x_i} \left(\left| \frac{\partial u}{\partial x_i} \right|^{p-2} \frac{\partial u}{\partial x_i} \right) + u^{1+\alpha}, \quad x \in \Omega, \quad t > 0, \quad (1)$$

$$u(x, 0) = u_0(x), \quad x \in \Omega, \quad (2)$$

$$u(x, t) = 0, \quad x \in \partial\Omega, \quad t \geq 0, \quad (3)$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain. This problem was proposed by Lions in [2]. Tsutsumi in [6] dealt with the Eq. (1) by considering following the problem:

$$\frac{\partial u}{\partial t} = \sum_{i=1}^n \frac{\partial}{\partial x_i} \left(\left| \frac{\partial u}{\partial x_i} \right|^{p-2} \frac{\partial u}{\partial x_i} \right) + \varphi(u). \quad (4)$$

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