Comments on "Multipartite Entanglement in Four-qubit Graph States"

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Abstract

The following comments are based on the article by M. Jafarpour and L. Assadi [Eur. Phys. J. D 70, 62 (2016), doi:10.1140/epjd/e2016-60555-5] which by means of Scott measure (or generalized Meyer-Wallach measure) the entanglement quantity of four-qubit graph states has been calculated. We are to reveal that the Scott measure (Q_m) nominates limits for m which would prevent us from calculating Q_3 in four-qubit system. Incidentally in a counterexample we will confirm as it was recently concluded in the mentioned article, the Q_2 quantity is not necessarily always greater than Q_3 in all the graph states.

Keywords: Entanglement, Graph states, Qubit

Recently, M. Jafarpour and L. Assadi [1] based on Scott measure have calculated the entanglement quantity in non-trivial four-qubit graphs. Scott studied various interesting aspects of *N*-qubit entanglement measures given by [2, 3]:

$$Q_m(|\psi\rangle) = \binom{N}{m}^{-1} \sum_{|S|=m} \frac{2^m}{2^m - 1}\left(1 - \mathrm{Tr}\left[\rho_S^2\right]\right), \qquad (1)$$

Where $S \subset \{1,\cdots,N\}$ and $\rho_S = \mathrm{Tr}_{\bar{S}}\left(|\psi\rangle\langle\psi|\right)$ is the reduced density matrix for S qubits after tracing out the rest. Also $m = 1,\cdots,\lfloor \frac{N}{2} \rfloor$ and $\lfloor \frac{N}{2} \rfloor$ is the integer part of $\frac{N}{2}$. The Q_m quantities $(0 \le Q_m \le 1)$ correspond to the average entanglement between subsystems that consists m qubits and the remaining $N-m$ qubits [4]. Meanwhile, Q_m is invariant under local unitary (LU) transformations, non-incremental on average under local operations and classical communication (LOCC). Hence on account of four-qubit system, we are only authorized to merely calculate Q_1 and Q_2. We have obtained $Q_1 = 1$ for all non-trivial four-qubit graphs (No. 1-41). Whereas the authors have calculated Q_3 in Table 1, leading to an incorrect result. Thus Section $6-d$ (Conclusions and discussion) leads to Q_2 being always greater than Q_3 in all the graph states. We will rectify in a counterexample their achieved result is incorrect in general. To clarify, take graph G_* for example, which is plotted in Figure 1. The graph state corresponding to graph G_* is as followed:

$$|G_*\rangle = \frac{1}{8}\big(|0,0,0\rangle|\phi_1\rangle + |0,0,1\rangle|\phi_2\rangle + |0,1,0\rangle|\phi_3\rangle + |0,1,1\rangle|\phi_4\rangle + |1,0,0\rangle|\phi_5\rangle + |1,0,1\rangle|\phi_6\rangle + |1,1,0\rangle|\phi_7\rangle + |1,1,1\rangle|\phi_8\rangle\big). \qquad (2)$$

Where:

$$\begin{aligned}
|\phi_1\rangle &= \{|0,0,0\rangle + |0,0,1\rangle + |0,1,0\rangle - |0,1,1\rangle + |1,0,0\rangle - |1,0,1\rangle - |1,1,0\rangle - |1,1,1\rangle\},\\
|\phi_2\rangle &= \{|0,0,0\rangle + |0,0,1\rangle + |0,1,0\rangle - |0,1,1\rangle - |1,0,0\rangle + |1,0,1\rangle + |1,1,0\rangle + |1,1,1\rangle\},\\
|\phi_3\rangle &= \{|0,0,0\rangle + |0,0,1\rangle - |0,1,0\rangle + |0,1,1\rangle + |1,0,0\rangle - |1,0,1\rangle + |1,1,0\rangle + |1,1,1\rangle\},\\
|\phi_4\rangle &= \{|0,0,0\rangle - |0,0,1\rangle + |0,1,0\rangle - |0,1,1\rangle + |1,0,0\rangle - |1,0,1\rangle + |1,1,0\rangle + |1,1,1\rangle\},\\
|\phi_5\rangle &= \{|0,0,0\rangle - |0,0,1\rangle + |0,1,0\rangle + |0,1,1\rangle + |1,0,0\rangle + |1,0,1\rangle - |1,1,0\rangle + |1,1,1\rangle\},\\
|\phi_6\rangle &= \{|0,0,1\rangle - |0,0,0\rangle - |0,1,0\rangle - |0,1,1\rangle + |1,0,0\rangle + |1,0,1\rangle - |1,1,0\rangle + |1,1,1\rangle\},\\
|\phi_7\rangle &= \{|0,0,1\rangle - |0,0,0\rangle + |0,1,0\rangle + |0,1,1\rangle - |1,0,0\rangle - |1,0,1\rangle - |1,1,0\rangle + |1,1,1\rangle\},\\
|\phi_8\rangle &= \{|0,0,1\rangle - |0,0,0\rangle + |0,1,0\rangle + |0,1,1\rangle + |1,0,0\rangle + |1,0,1\rangle + |1,1,0\rangle - |1,1,1\rangle\}.
\end{aligned} \qquad (3)$$