



The Investigation of the existence, behavior, and multiplicity of solutions for $-p$ Laplacian boundary value problems under the Dirichlet boundary conditions

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Abstract:

Expression and description of most physical and engineering phenomena such as wave propagation, heat transfer, magnetism, etc. lead to the investigation and equational solution of hyperbolic, parabolic, and elliptic kinds. These equations are from basic topics of boundary value and initial value problems. In this paper, we investigate the problem of quasi-linear boundary:

$$-(|u'|^{p-2}u')' = \lambda f(u) \quad \text{in } (0,1)$$

$$u(0) = u(1) = 0$$

Where p and λ are two actual parameters and $p > 1$, $\lambda > 0$ and f are a nonlinear third-order function.

To obtain the number of solutions of the above problem, we use quadrature method.

Keywords:

P -Laplacian, multiplicity of results, third-order non-linear functions, quadrature method and time mapping.

Introduction

The existence and multiplicity of the solutions of boundary value problems involving the p -Laplacian operator has attracted the attention of many authors. Especially the p -Laplace operator:

$$\Delta_p(u) = \operatorname{div}(|\nabla u|^{p-2}\nabla u), \quad p > 1$$

In the last decade, much attention has been paid to the fact, because many physical models include models of this operator. In this paper, the

boundary value problem consists of a general one-dimensional p -Laplacian:

$$-(\alpha(|t|^P)|t|^{P-2}t)' = f(u) \quad \text{in } (0,1)$$

$$u(0) = u(1) = 0 \quad (1)$$

When $P > 1$ and $\alpha \in (R^+, R)$ is not necessarily equal to 1 and nonlinear function is continuous and odd is on attention. In such a way that there are not $t_0 \geq 0$ و $\theta \in (0, \frac{1}{P})$ invariants and we will have:

$$\theta t f(t) \geq \int_0^t f(\tau) d\tau > 0, \quad \forall |t| \geq t_0 \quad (2)$$

Under some conditions, we show on α that there is $k_0 \in N$ in such a way that problem 1 accept at least two weak solutions in A_k for each integer $k \geq k_0$. Also for $|t|$ s big enough, it is concluded from (2) that:

$$f(t) > 0 \quad (3)$$

We will restrict this case to $\alpha(t) = 1$ and $\forall t \geq 0$ and we will see that f is an odd function which is valid in the inverse inequality (3).

In this case, the boundary value problem:

$$-(|u'|^{p-2}u')' = \lambda f(u) \quad \text{in } (0,1)$$

$$u(0) = u(1) = 0$$

When $\lambda > 0$ and $P > 1$ are two real parameters and we considered $f \in C(R, R)$ as a third-order nonlinear function. While f is similar to the third-order polynomial function:

$u \rightarrow u(\alpha^2 - u^2)$ has $\alpha > 0$ for some. In order to investigate this problem, we use the time