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Error growth in the numerical integration of periodic orbits☆

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Abstract

This paper is concerned with the long term behaviour of the error generated by one step methods in the numerical integration of periodic flows. Assuming numerical methods where the global error possesses an asymptotic expansion and a periodic flow with the period depending smoothly on the starting point, some conditions that ensure an asymptotically linear growth of the error with the number of periods are given. A study of the error growth of first integrals is also carried out. The error behaviour of Runge–Kutta methods implemented with fixed or variable step size with a smooth step size function, with a projection technique on the invariants of the problem is considered.

Keywords: Geometric integration; Runge–Kutta methods; Invariant preservation; Long-time integration

1. Introduction

Although differential systems with periodic flows are mathematically rather exceptional, they have been widely used in the design and test of many numerical integrators. The elliptic Kepler problem is a well known example of a periodic flow frequently used as a test problem for standard integrators because one can get easily information about the error behaviour of the solution and first integrals along many periods of integration. Furthermore, as remarked by Stuart and Humphries in [17], in order to design algorithms that provide a reliable qualitative information on dynamical systems, it is important to understand the behaviour of the numerical simulations and, since periodic orbits play an important role in this study, it is of interest to study their behaviour under discretizations. In particular, in Chapter 8 some results on preservation properties in Hamiltonian and related problems are studied.

An earlier contribution to the study of the error behaviour of numerical integrators on periodic flows appears in the paper of Calvo and Sanz-Serna [5] which deals with Kepler's problem. Here the authors explain the asymptotically linear error growth with the number of periods provided by symplectic algorithms applied to Kepler's problem with fixed step sizes. Later, Cano and Sanz-Serna [6] have considered the more general problem of the error growth of one step methods in the numerical integration of a (possibly isolated) periodic orbit. There, under conditions that ensure the existence of an asymptotic expansion of the global error in powers of the step size, it is found that the long term behaviour of the coefficients of the global error is determined by the powers of the monodromy matrix of the periodic

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