Available online at www.sciencedirect.com

ScienceDirect

Mathematics and Computers in Simulation 81 (2011) 980–995

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Reduction to invariant cones for non-smooth systems

T. Küpper *, H.A. Hosham

Mathematisches Institut, Universität zu Köln, Weyertal 86-90, Germany

Available online 21 October 2010

Abstract

The reduction of smooth dynamical systems to lower dimensional center manifolds containing the essential bifurcation dynamics is a very useful approach both for theoretical investigations as well as for numerical computation. Since this approach relies on smoothness properties of the system and on the existence of a basic linearization the question arises if this approach can be carried over to non-smooth systems. Extending previous works we show that such a reduction is indeed possible by using an appropriate Poincaré map: the linearization will be replaced by a basic piecewise linear system; a fixed point of the Poincaré map generates an invariant cone which takes the role of the center manifold. The occurrence of nonlinear higher order terms will change this invariant "manifold" to a cone-like surface in $\mathbb{R}^n$ containing the essential dynamics of the original problem. In that way the bifurcation analysis can be reduced to the study of one-dimensional maps.

Keywords: Non-smooth dynamical systems; Invariant cones; Periodic orbits; Generalized Hopf bifurcation

1. Introduction

The center manifold approach provides a powerful tool to reduce high-dimensional parameter dependent dynamical systems to lower dimensional systems carrying the essential dynamics responsible for example for bifurcation processes. In this paper we will continue to investigate if a similar approach is available for non-smooth systems. First we briefly recall the key facts for smooth systems.

Let

$$\dot{\xi} = f(\xi, \lambda), \quad (\xi \in \mathbb{R}^n, \lambda \in \mathbb{R}^p), \tag{1}$$

denote a smooth dynamical system with stationary solution $\bar{\xi} = 0$.

Using linearization and transformations according to the structure of the eigenvalues of the linearization $A:=(\partial f/\partial \xi)(\bar{\xi})$ of f, Eq. (1) can be stated in the following form with $\xi = (x, y, z)^T$ and an accordingly arranged matrix A

$$A = \begin{pmatrix} A^- & 0 & 0 \\ 0 & A^0 & 0 \\ 0 & 0 & A^+ \end{pmatrix},$$

* Corresponding author.
E-mail addresses: kuepper@math.uni-koeln.de (T. Küpper), hbakit@math.uni-koeln.de (H.A. Hosham).

0378-4754/$36.00
doi:10.1016/j.matcom.2010.10.004