Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 81 (2011) 1068–1084
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original article

Trigonometrically fitted two-step hybrid methods for special second order ordinary differential equations

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Available online 21 October 2010

Abstract

The purpose of this paper is to derive two-step hybrid methods for second order ordinary differential equations with oscillatory or periodic solutions. We show the constructive technique of methods based on trigonometric and mixed polynomial fitting and consider the linear stability analysis of such methods. We then carry out some numerical experiments underlining the properties of the derived classes of methods.

Keywords: Two-step hybrid methods; Trigonometrical fitting; Second order ordinary differential equations

1. Introduction

It is the aim of our paper to derive new classes of numerical methods for solving initial value problems based on second order ordinary differential equations (ODEs)

$$\begin{cases} y'' = f(x, y), \\ y'(x_0) = y'_0, \\ y(x_0) = y_0 \end{cases} \tag{1}$$

with f smooth enough in order to ensure the existence and uniqueness of the solution. Although problem (1) can be solved by transforming it into a system of first order ODEs of double dimension, the development of numerical methods for its direct integration seems more natural and efficient. This problem, having periodic or oscillatory solutions, often appears in many applications: celestial mechanics, seismology, molecular dynamics, and so on (see for instance [23,26] and references therein contained). Classical numerical methods for ODEs relied on polynomials may not be very well-suited to periodic or oscillatory behaviour. In the framework of exponential fitting many numerical methods have been adapted in order to exactly integrate basis of functions other than polynomials, for instance the exponential basis (see [16] and references therein contained), in order to catch the oscillatory behaviour. The parameters of these methods depend on the values of frequencies, which appear in the solution. In order to adapt the collocation technique [14,18] to an oscillatory behaviour, the collocation function has been chosen as a linear combination of trigonometric functions

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doi:10.1016/j.matcom.2010.10.011