Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 81 (2011) 2103–2128
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original article

A convergence of a MFE-FV method for immiscible compressible flow in heterogeneous porous media

Mustapha El Ossmani

Universit Moulay Ismaïl, ENSAM-Meknès, Equipe EMMACS, Marjane II, B.P. 4024, Meknès, 50 000, Morocco

Received 19 January 2010; received in revised form 22 August 2010; accepted 8 December 2010
Available online 31 December 2010

Abstract

This paper deals with the development and analysis of a numerical method for a coupled system describing immiscible compressible two-phase flow through heterogeneous porous media. The system is modelled in a fractional flow formulation which consists of a parabolic equation (the global pressure equation) coupled with a nonlinear degenerated diffusion-convection one (the saturation equation). A mixed finite element (MFE) method is used to discretize the pressure equation and is combined with a conservative finite volume (FV) method on unstructured grids for the saturation equation. It is shown that the FV scheme satisfies a discrete maximum principle. We derive L^∞ and BV estimates under an appropriate CFL condition. Then we prove the convergence of the approximate solution to a weak solution of the coupled system. Numerical results for water-gas flow through engineered and geological barriers for a geological repository of radioactive waste are presented to illustrate the performance of the method in two space dimensions.

Keywords: Finite volume method; Mixed finite element; Immiscible compressible flow; Porous media; Nuclear waste

1. Introduction

Motivation for the mathematical problem treated herein arises from the area of modeling two-phase flow problems related to the environment and the energy fields. Many difficult issues appear in the numerical simulation of complex fluid processes in reservoir engineering, subsurface contaminant transport and remediation, sequestration of CO_2 and other applications. The mathematical models commonly used to describe these processes are coupled systems of partial differential equations (PDEs), which are basically of advection-diffusion type. However, the simulation of realistic problems requires in some case the combination of different numerical methods.

In this paper, we focus our attention on the of immiscible compressible two-phase flow in porous media, in the framework of the geological disposal of radioactive waste. As a matter of fact, one of the solutions envisaged for managing waste produced by nuclear industry is to dispose it in deep geological formations chosen for their ability to prevent and attenuate possible releases of radionuclides in the geosphere. In the frame of designing nuclear waste geological repositories appears a problem of possible two-phase flow of water and gas. Multiple recent studies have established that in such installations important amounts of gases are expected to be produced in particular due to the corrosion of metallic components used in the repository design. The creation and transport of a gas phase is an issue of

E-mail address: mustapha.elossmani@gmail.com

0378-4754/$36.00
doi:10.1016/j.matcom.2010.12.007