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Original article

A comparative linear mean-square stability analysis of Maruyama- and Milstein-type methods

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Abstract

In this article we compare the mean-square stability properties of the θ-Maruyama and θ-Milstein method that are used to solve stochastic differential equations. For the linear stability analysis, we propose an extension of the standard geometric Brownian motion as a test equation and consider a scalar linear test equation with several multiplicative noise terms. This test equation allows to begin investigating the influence of multi-dimensional noise on the stability behaviour of the methods while the analysis is still tractable. Our findings include: (i) the stability condition for the θ-Milstein method and thus, for some choices of θ, the conditions on the step-size, are much more restrictive than those for the θ-Maruyama method; (ii) the precise stability region of the θ-Milstein method explicitly depends on the noise terms. Further, we investigate the effect of introducing partial implicitness in the diffusion approximation terms of Milstein-type methods, thus obtaining the possibility to control the stability properties of these methods with a further method parameter σ. Numerical examples illustrate the results and provide a comparison of the stability behaviour of the different methods.

AMS Classification: 60H10; 65C20; 65U05; 65L20

Keywords: Stochastic differential equations; Asymptotic mean-square stability; θ-Maruyama method; θ-Milstein method; Linear stability analysis

1. Introduction

In recent years the area of numerical analysis of stochastic differential equations (SDEs) has expanded at a fast pace. This interest has been driven by different application areas, such as computational finance, neuroscience or electrical circuit engineering. A large part of research in stochastic numerics has been aimed towards the development and strong and weak convergence analysis of several classes of numerical methods. A further important issue for the investigation of numerical methods consists of examining methods for their ability to preserve qualitative features of the continuous system they are developed to approximate. A linear stability analysis is usually the

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