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On optimal convergence rate of finite element solutions of boundary value problems on adaptive anisotropic meshes

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Abstract

We describe a new method for generating meshes that minimize the gradient of a discretization error. The key element of this method is construction of a tensor metric from edge-based error estimates. In our papers [1–4] we applied this metric for generating meshes that minimize the gradient of P_1-interpolation error and proved that for a mesh with N triangles, the L^2-norm of gradient of the interpolation error is proportional to $N^{-1/2}$. In the present paper we recover the tensor metric using hierarchical a posteriori error estimates. Optimal reduction of the discretization error on a sequence of adaptive meshes will be illustrated numerically for boundary value problems ranging from a linear isotropic diffusion equation to a nonlinear transonic potential equation.

Keywords: Metric-based adaptation; Finite element method; Quasi-optimal meshes

1. Introduction

Generation of adaptive meshes is the active research area. The paper is devoted to generation of meshes which minimize the energy norm of finite element solutions to boundary value problems. The minimum is sought over the set of conformal triangulations with a fixed number of simplexes. An approximate solution of the minimization problem is deemed sufficient if (a) the discretization error is close to that on the optimal mesh and (b) the error reduction rate on a sequence of generated meshes is optimal. We call such meshes *quasi-optimal*.

In [1–4], we analyzed the problem of minimizing the L^p-norm of the gradient of the interpolation error and proposed a numerical method for generation of d-dimensional quasi-optimal meshes. The method recovers a tensor metric inside each simplex from function values at the simplex vertices and edge midpoints. The analysis is based on a geometric representation of the error and a relaxed saturation assumption. In [1] we applied the method to finite element discretizations of linear elliptic PDEs. The numerical experiments have shown optimal reduction of hierarchical error estimates on anisotropic meshes. In this paper, we analyze numerically robustness

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