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Mathematics and Computers in Simulation 81 (2011) 1572–1592

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Original article

Convergence conditions for iterative methods seeking multi-component solitary waves with prescribed quadratic conserved quantities

T.I. Lakoba *

Department of Mathematics and Statistics, 16 Colchester Ave., University of Vermont, Burlington, VT 05401, USA

Received 26 December 2009; received in revised form 16 October 2010; accepted 4 November 2010
Available online 7 December 2010

Abstract

We obtain linearized (i.e., non-global) convergence conditions for iterative methods that seek solitary waves with prescribed values of quadratic conserved quantities of multi-component Hamiltonian nonlinear wave equations. These conditions extend the ones found for single-component solitary waves in a recent publication by Yang and the present author. We also show that, and why, these convergence conditions coincide with dynamical stability conditions for ground-state solitary waves.

Notably, our analysis applies regardless of whether the number of quadratic conserved quantities, s, equals or is less than the number of equations, S. To illustrate the situation when $s < S$, we use one of our iterative methods to find ground-state solitary waves in spin-1 Bose–Einstein condensates in a magnetic field ($s = 2$, $S = 3$).

Keywords: Coupled nonlinear wave equations; Solitary waves; Iterative methods; Spinor Bose–Einstein condensates

1. Introduction and background

Solitary wave solutions of most nonlinear wave equations can be found only numerically. Recently, Yang and the present author obtained [22] conditions under which an iterative numerical method can converge to stationary solitary waves of single-component Hamiltonian nonlinear wave equations. When this method, in what follows referred to as the imaginary-time evolution method (ITEM), converges, it provides one with a numerical approximation of a solitary waves with a prescribed value of a quadratic conserved quantity usually referred to either as *power* or the number of particles. However, many phenomena are described not by a single equation but by systems of coupled equations. Therefore, it is of interest to obtain conditions under which a multi-component counterpart of the ITEM would be guaranteed to converge to a multi-component solitary wave. We obtain such a condition in this work. Moreover, generalizing an observation made in [22], we show that the multi-component ITEM converges only to those ground states of nonlinear wave equations which are dynamically stable, and explain why this is the case.

* Tel.: +1 802 656 2610.
E-mail address: lakobati@cems.uvm.edu

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doi:10.1016/j.matcom.2010.11.008