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Original article

Border collision bifurcations in one-dimensional linear-hyperbolic maps

Laura Gardini [a], Fabio Tramontana [a,*], Iryna Sushko [b]

[a] Università degli Studi di Urbino, Department of Economics and Quantitative Methods, Via Saffi 42, 61029 Urbino, Italy
[b] Institute of Mathematics, National Academy of Sciences of Ukraine, and Kiev School of Economics, Kiev, Ukraine

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Abstract

In this paper we consider a continuous one-dimensional map, which is linear on one side of a generic kink point and hyperbolic on the other side. This kind of map is widely used in the applied context. Due to the simple expression of the two functions involved, in particular cases it is possible to determine analytically the border collision bifurcation curves that characterize the dynamic behaviors of the model. In the more general model we show that the steps to be performed are the same, although the analytical expressions are not given in explicit form.

Keywords: Piecewise smooth map; Border-collision bifurcation; Bistability

1. Introduction

In recent years a significant amount of papers have been published regarding piecewise smooth (PWS) systems because of their wide use in applications. Several models in engineering and physical sciences are described by PWS systems (see [1,3,6–8,12–14,16,17,24,25,30,38]), as well as in economics (see [28,29]). The first results on PWS systems date back to several years ago, not only for continuous models but also for discontinuous systems [2,18,19,23], and are extended also to two-dimensional models (see [5,31,34,39,40]).

The main point in the analysis of PWS systems is the occurrence of *border collision bifurcations* (BCB), due to the merging (or collapse) of some invariant set (a fixed point, a periodic point of a cycle, or the boundary of any invariant set) with the kink point in which the function changes its definition. This may lead to a drastic change, unexpected (i.e. impossible) in the framework of smooth systems. Such border collision bifurcations are responsible, for example, for the direct transition from regular regime to chaotic dynamics, or to divergence [6,20–22,26,27].

The analysis of the effect of a collision of an invariant set with a kink point, a boundary for the map definition, in general is not an easy task. It is a bifurcation which depends on the shape of the map on the two sides of the collision, and may lead to several different dynamic effects. For example, the dynamics can change suddenly from an attracting fixed point to an attracting cycle of any period, or to chaotic dynamics (true chaos or strict chaos following [11], for a full measure chaotic set, and robust chaos following [4], because the chaotic set is persistent as a function of the parameters).

* Corresponding author.
E-mail addresses: laura.gardini@uniurb.it (L. Gardini), f.tramontana@univpm.it (F. Tramontana), sushko@imath.kiev.ua (I. Sushko).

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